Development of Control Laws for an AFCS of a Helicopter

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by

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CERTIFICATE

This is to certify that the work contained in the thesis entitled Development of Control Laws for an AFCS of a Helicopter by Geetha K Thampi, has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

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Nomenclature

u_h, v_h, w_h	Traslational velocity components of helicopter along fuselage x-, y- and z-axes.
p, q, r	Angular velocity components of helicopter about fuselage x-, y- and z-axes.
ϕ, phi	Euler roll angle(roll right positive).
θ, theta	Euler pitch angle(nose up positive).
ψ, psi	Euler yaw angle(nose right positive).
θ_{1s}	Longitudinal cyclic pitch(negative forward).
θ_{1c}	Lateral cyclic pitch(positive right).
R	Field of complex numbers.
H_∞	Hardy space.
$\|\cdot\|_\infty$	Norm on H_∞.

Chapter 1

Introduction

In the modern era, helicopters have established themselves as unique flying vehicles due to their flexible manoeuvrability and growing utility in both civil and military applications. However, helicopters are basically unstable systems and necessiate the pilots to fly them by continuously applying the control input. This causes an enormous increase in the pilot's work load and leads to poor riding qualities in response to wind gust and poor visibility. In addition, in military helicopters this severly affects the performance of weapon delivery. Due to these reasons there is a strong demand for designing an Automatic Flight Control System(AFCS) for helicopters to improve their stability and control characteristics, and to ensure safety in flight as well as good flying performance. In short, the objective of any AFCS is to provide the pilot with a means to control the vehicle safely and effectively throughout the flight envelope, ie, to provide good Handling Qualities(HQ). The design and development of AFCS has evolved over the past four decades from simple mechanical to complex sophisticated digital fly-by-wire flight control systems. All rotorcraft today employ some form of feedback control. This may be in the form of an autopilot, a limited authority control and stability augmentation system, a full authority fly-by-wire system or even a fully integrated centralised vehicle management system. Typical functions provided by such an AFCS are

- Control and stability augmentation for superior Handling Qualities.
- Automatic flight trajectory and forward speed control.
- Wind gust response reduction for extended service of life; ride quality enhancement for increased crew and pilot comfort.

- Aerodynamic and propulsion performance optimization for reduced operating costs.

- Flutter suppression and failure reconfiguration for increased safety.

Since it is very difficult to incorporate all the above functions in an AFCS reliably, most of the rotorcraft are fitted with AFCS which can perform the basic functions such as (1) Stability and Control augmentation. (2)Autopilot functions such as regulation of attitude, speed and flight path. (3)Signalling or annunciation of mission cues. The most important requirement is that the AFCS must provide the desired functions at all weather conditions without compromising safety of flight.

Over the last two decades, AFCS requirements have been in a state of transition. Rotarywing transmission systems have become more demanding, requiring vehicle management systems capable of conducting highly aggressive mission under night or adverse weather conditions. Also the performance needs of the modern helicopter requires exceptional agility, manoeuvrability and handling characteristics. This promotes the need for control systems of increasing complexity in order to provide acceptable HQ throughout the operational environment. The implementation of highly augmented AFCS, new controller types and pilot information systems will give the designer more flexibility to alter the response characteristics of the overall helicopter system and to tailor the desired flying qualities.

On the theoretical front, powerful new techniques for design of multivariable control systems such as factorization theory, H_∞ control, μ synthesis or LQG based H_∞ control have emerged in the last ten years. These have been succesfully applied to a variety of control problems in engineering. In particular, the popularity of the H_∞ control methods [1, 6, 7] are primarily attributed to features such as (i) possibility of combining multivariable frequency response and sensitivity properties; (ii) guidelines for transforming realistic design requirements into performance specifications and (iii) availability of state space theoretic algorithms and the associated efficient numerical computational algorithms.

1.1 Problem addressed in the thesis

In this thesis, a limited but realistic and comprehensive design study is carried out, on a helicopter model, using the techniques of H_∞ optimization and factorization theory. More specifically the thesis addresses the formulation and solution of the following problems for a realistic helicopter model.

1. Design of closed loop control laws for stability augmentation satisfying the MIL-H-8501A specification, for two distinct flight conditions, namely at hover and at a forward speed of 200kph.

2. Control augmentation compensator design satisfying the MIL-H-8501A specification for both of the above flight conditions.

3. Robust closed loop stabilization at hover.

Any rotorcraft is a highly coupled multivariable plant. Hence the helicopter model provides an interesting and challenging case study particularly for achieving performance specification of a standard type such as MIL-H-8501A. Now performance specification like ADS-33 have also begun to be studied from the point of view of control law design. In this thesis however, we shall resrict ourselves to the MIL standard.

1.2 Organization of the thesis

The chapters in the rest of the thesis are organized as follows. Chapter 2 discusses the dynamics of the helicopter, basic flight controls, major coupling effects present in the system etc. It also gives an outline of the HQ performance specifications and describes how the flight control system design poses itself as an interesting problem for the control engineer.

In chapter 3, a procedure for linear design is drawn up based on HQ requirements. For the two flight conditions namely, hover and forward speed of 200kph, two separate two degrees of freedom controllers are designed. These include a stabilizing closed loop controller for achieving closed loop poles as per MIL-H-8501A specification and a forward gain controller for augmenting the pilot's input response. Finally, a robust stabilizing closed loop control law is also designed for hover by considering the multiplicative plant

unceraint model. This design is carried out using the modern state space algorithmic procedure for the H_∞ controller design.

In Chapter 4 the simulation of the final control laws are carried out and the required performances are verified. A full discussion of all related simulations is also carried out.

Chapter 5 summarizes the conclusions and presents a discussion on the scope for future work.

1.3 Survey of literature

Various methods are well known for the design of multivariable flight control systems. One of the most popular methods uses classical single input single output(SISO) techniques to design one control loop at a time. For instance, the Apache AV05 YAH-64 [5], was designed using single loop techniques, with proportional-plus-integral(PI) configuration. Eventhough it gave satisfactoty results, this method can be useful for certain problems only. Its capability is extremely limited for highly coupled multivariable systems. Furthermore, analysis of multivariable systems with SISO techniques can often give misleading results. There are two main methods of design of control laws for a multivariable system namely, Riccati-based methods such as H_2, H_∞ and structural singular value. Eventhough the choice of the design method is not clear, the Riccati-based solution methods have a broader acceptability. Doyle et al [3], has pioneered the state space approach to H_2 and H_∞ control problems which has found many applications in recent times. The reference [15] discusses the control law design using H_∞ methods, of a multivariable controller to give decoupled pitch and depth control of a submarine. In addition to the basic multivariable design, the paper also considers the development of a controller conditioning strategy to ensure that performance is maintained in the face of actuator saturation. The simulation of controller and desaturation strategy in this paper, using the H_∞ techniques shows very satisfactory performance.

The application of multivariable design techniques such as Direct Nyquist Array and Characteristic Loci method, to achieve decoupled control of the longitudinal dynamics of a Vertical/Short Take-off and Landing(VSTOL) aircraft is described in reference [15]. It was demonstrated that the characteristic loci method was difficult to apply succesfully in this application. Both performance and robustness were not satisfactory. Work is in

progress to apply H_∞ methods to the VSTOL problem.

In reference [15], a design example of how H_∞ optimal control can be scheduled using a controller switching strategy for a VSTOL aircraft is given. The linear design makes use of an open loop singular value shaping method. The technique used assigns each linear controller design to a portion of the operational envelope.

Inorder to investigate the handling, control and display requirements for future VSTOL aircraft, the UK Ministry of Defence is sponsoring a research programme called Vectored thrust Aircraft Advanced flight Control(VAAC). The research is extending the database used to derive the handling qualities requirements for conventional take-off and landing configuration to future STOVL aircraft. The flight control and handling research with the VAAC Harrier is given in [12].

A collaborative research programme has been initiated between Royal Aerospace Establishment(RAE) Bedford, Leicester University and Cambridge Control to build on the work done by Yue and Postlethwaite [17]. Yue and Postlethwaite, in [17] presented an application of H_∞ optimization theory to the design of helicopter control system. The controller designed was evaluated at RAE, Bedford and the results showed that this controller conforms to the military handling qualities specification. According to the literature, the piloted simulation results were excellent showing that it was feasible to use an H_∞ controller for improving the handling qualities of a typical high performance helicopter.

Reference [11] presents the synthesis and analysis of multivariable robust control systems for a UH-60 Black Hawk helicopter in forward flight based on the ADS-33 specification and the structured singular value analysis. Several robust controllers with different configurations were constructed to determine which configuration provides the best overall HQ performance.

Despite the emergence of the extensive control design research programmes for helicopters in the Western countries, similar published work in India is almost absent. This thesis is one of the modest attempts at beginning an indigeneous effort in control system design program for helicopters.

Chapter 2

Helicopter Dynamics

2.1 Introduction

The design of control system requires a thorough appreciation of the dynamics of the plant to be controlled.. Flight control system is no exception. So, for designing an AFCS for a helicopter and evaluating its performance, it is essential to understand the fundamentals of helicopter dynamics and its control. The basic function of AFCS design is to provide a stable and well behaved platform for undertaking the required mission. Helicopters use rotating wings (rotors) to produce lift, propulsive power and control. The consequent versatility of flight envelope from vertical take- off/hovering to high speed flight explains their wide spread use and continuing efforts to improve their performance.

2.2 Basic flight controls

The flight control of a helicopter is achieved by providing the following four control inputs. They are (i) Collective pitch of the main rotor, denoted as θ_0, for increasing thrust or lift of the main rotor. (ii)Longitudinal cyclic pitch, denoted as θ_{1s}, for tilting the thrust vector in longitudinal direction. (iii)Lateral cyclic pitch, denoted as θ_{1c}, for tilting the thrust vector in lateral direction. (iv)Tail rotor collective pitch, denoted as θ_{0t}, for yaw control. See Fig.(2.1). These inputs are used as control inputs in any fly-by-wire system. Their functions are described in more detail below [4].

1. *Main rotor collective* :– By collective input, the pilot varies the average pitch angle of all the main rotor blades and thereby provides a method of controlling the total

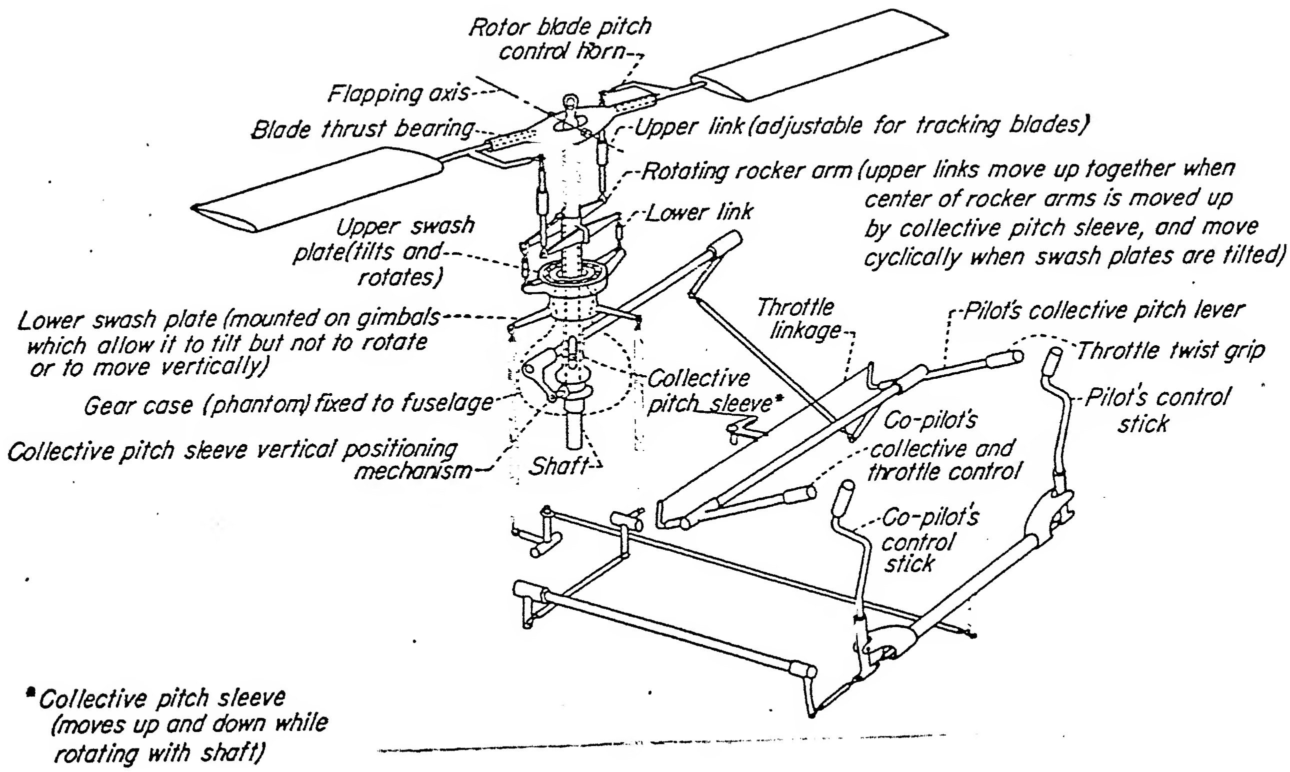

Figure 2.1: Control System of a Conventional Helicopter

rotor thrust and lift. The rotor speed is controlled by an independent engine control system, which governs the rotor speed. The engine contol system should act in co-ordination with main rotor collective because any increase in Angle of Attack(AOA) will also increase the drag on the blades tending to slow down the rotor. Since the rotor speed should remain constant, there should be a proportionate change in power to compensate for this change in drag. This control is carried out by the engine control system.

2. *Cyclic pitch controls* :– Cyclic pitch controls cause the rotor disc to tilt and change the direction of lift force without changing its magnitude. The rotor is tilted by

changing the pitch of the rotor blades individually once during the cycle of rotation.

3. *Tail rotor collective* :- The main purpose of the tail rotor is to counteract the main rotor torque. In the absence of the counteracting torque, the helicopter has completely unstable attitude. This control is used differently between the hover and forward flight. In hovering it is used to rotate the helicopter about a vertical axis and control heading. In forward flight it is used to ensure co-ordinated flight. ie, to prevent side slip.

Mathematically, the control input to k-th blade of the main rotor can be expressed as

$$\theta_k = \theta_0 + \theta_{1c}\, cos\psi_k + \theta_{1s}\, sin\psi_k$$

where $\psi_k = \frac{2\pi}{N}(k-1),\ k = 1 \ldots N$

ψ_k represents the azimuth location of the k-th blade and N is the total number of blades in the main rotor system.

2.3 Major coupling effects

In almost all flight conditions, the uncompensated helicopter is unstable so the pilot is required to make continual small adjustments to maintain stability, even in a trimmed state. In addition a number of interactions are present.

1. Main rotor collective – Tail rotor :- A change in main rotor collective always requires a change in tail rotor collective to balance the change in torque and a corresponding change in power is required to maintain rotor speed.

2. Cyclic -Collective :- When longitudinal or lateral cyclic is applied to tilt the rotor disk, to increase forward speed or to roll, to initiate a turn, the pilot must increase the collective to maintain height. This is because tilting of the thrust vector reduces its vertical component.

3. Longitudinal Cyclic - Lateral Cyclic :- Due to gyroscopic effects of the rotor system, pitching and rolling degrees of freedom of the helicopter are coupled. The moment of inertia of a helicopter in roll is about one fourth of that in pitch. Consequently, a longitudinal cyclic input requires significant correction with the lateral cyclic.

2.4 Equations of motion

The equations of motion of a rigid helicopter can be written in the body fixed coordinate system. The rigid body has both translational and rotational motion. The dynamical equations of motion of a helicopter is given by [10]

$$
\begin{aligned}
\dot{u}_h &= -(w_h q - v_h r) + X/M_a - g\ sin\theta \\
\dot{v}_h &= -(u_h r - w_h p) + Y/M_a + g\ cos\theta\ sin\phi \\
\dot{w}_h &= -(v_h p - u_h q) + Z/M_a + g\ cos\theta\ cos\phi \\
I_{xx}\dot{p} &= (I_{yy} - I_{zz})qr + I_{xz}(\dot{r} + pq) + L \\
I_{yy}\dot{q} &= (I_{zz} - I_{xx})rp + I_{xz}(r^2 - p^2) + M \\
I_{zz}\dot{r} &= (I_{xx} - I_{yy})pq + I_{xz}(\dot{p} - qr) + N
\end{aligned}
\tag{2.1}
$$

$$
\begin{aligned}
\dot{\phi} &= p + q\ sin\phi\ tan\theta + r\ cos\phi\ tan\theta \\
\dot{\theta} &= q\ cos\phi - r\ sin\phi \\
\dot{\psi} &= q\ sin\phi\ sec\theta + r\ cos\phi\ sec\theta
\end{aligned}
$$

The states of the system and control inputs chosen are

$$
\begin{aligned}
x^T &= [u_h,\ v_h,\ w_h,\ p,\ q,\ r,\ \theta,\ \phi,\ \psi] \\
u^T &= [\theta_0,\ \theta_{1s},\ \theta_{1c},\ \theta_{0t}]
\end{aligned}
$$

where u_h, v_h, w_h are the translational velocities along the three orthogonal directions of the fuselage fixed axis system. p, q, r are the angular velocities about the same axis and θ, ϕ, ψ are the Euler angles, defining the orientation of the body axes relative to the earth. The control vector has four components: main rotor collective(θ_0), longitudinal cyclic(θ_{1s}), lateral cyclic(θ_{1c}) and tail rotor collective(θ_{0t}). Now, Eqn.(2.1) can be written in the form

$$
\dot{x} = F(x, u) \tag{2.2}
$$

where F is a nonlinear function of x and u. The solution of Eqn.(2.1) depends upon the initial conditions of the motion state vector and the time variation of the vector

function F(x,u), that includes aerodynamic loads, gravitational forces, inertial forces and moments. The trajectory can be computed using any of a number of different numerical integration schemes, but gives little insight in to the physics of flight behaviour. We need to turn to analytic solutions to obtain a deeper understanding between cause and effect. But the scope for deriving analytic solutions of general nonlinear differential equation is extremely limited. So we go for the linearised approximation of Eqn.(2.2). The essence of linearisation is the assumption that the motion can be considered as a perturbation about a trim condition. The function can usually be expanded in terms of the motion and control variables.

Similarly, the response can be written in the form

$$x = x_e + \delta x \tag{2.3}$$

where x_e is equilibrium value of the state vector which is referred to as trim and δx is the perturbation away from trim. By doing the linearisation of Eqn.(2.2) at x_e and trim control $u_e = 0$ the following equations are obtained where x denotes the perturbation δx at x_e.

$$\dot{x} = Ax + Bu_e \tag{2.4}$$

where the state matrix A is given by

$$A = \left(\frac{\partial F}{\partial x}\right)_{x=x_e,\ u=u_e}$$

and the control matrix B is given by

$$B = \left(\frac{\partial F}{\partial u}\right)_{x=x_e,\ u=u_e}$$

Thus by performing the linearisation of Eqn.(2.1), we will get the system and control matrices as given below. Denoting $\frac{\partial X}{\partial u} = X_u$ and with derivatives written in semi-

normalised form. ie, $X_u = X_u/M_a$.

$$
\mathbf{A} = \left[\begin{array}{c|c|c|c|c|c|c|c}
X_u & X_w - Q_e & X_q - W_e & -g\cos\Theta_e & X_v + R_e & X_p & 0 & X_r + V_e \\ \hline
Z_u + Q_e & Z_w & Z_q + U_e & -g\cos\Phi_e\sin\Theta_e & Z_v - P_e & Z_p - V_e & -g\sin\Phi_e\cos\Theta_e & Z_r \\ \hline
M_u & M_w & M_q & 0 & M_v & \begin{array}{c} M_p - 2P_e I_{xz}/I_{yy} \\ -R_e(I_{xx} - I_{zz})/I_{yy} \end{array} & 0 & \begin{array}{c} M_r + 2R_e I_{xz}/I_{yy} \\ -P_e(I_{xx} - I_{zz})/I_{yy} \end{array} \\ \hline
0 & 0 & \cos\Phi_e & 0 & 0 & 0 & -\Omega_a\cos\Theta_e & -\sin\Phi_e \\ \hline
Y_u - R_e & Y_w + P_e & Y_q & -g\sin\Phi_e\sin\Theta_e & Y_v & Y_p + W_e & g\cos\Phi_e\cos\Theta_e & Y_r - U_e \\ \hline
L'_u & L'_w & L'_q + k_1 P_e - k_2 R_e & 0 & L'_v & L'_p + k_1 Q_e & 0 & L'_r - k_2 Q_e \\ \hline
0 & 0 & \sin\Phi_e\tan\Theta_e & \Omega_a\sec\Theta_e & 0 & 1 & 0 & \cos\Phi_e\tan\Theta_e \\ \hline
N'_u & N'_w & N'_q - k_1 R_e - k_3 P_e & 0 & N'_v & N'_p - k_3 Q_e & 0 & N'_r - k_1 Q_e
\end{array}\right]
$$

$$
\mathbf{B} = \left[\begin{array}{c|c|c|c}
X_{\theta_0} & X_{\theta_{1s}} & X_{\theta_{1c}} & X_{\theta_{0T}} \\ \hline
Z_{\theta_0} & Z_{\theta_{1s}} & Z_{\theta_{1c}} & Z_{\theta_{0T}} \\ \hline
M_{\theta_0} & M_{\theta_{1s}} & M_{\theta_{1c}} & M_{\theta_{0T}} \\ \hline
0 & 0 & 0 & 0 \\ \hline
Y_{\theta_0} & Y_{\theta_{1s}} & Y_{\theta_{1c}} & Y_{\theta_{0T}} \\ \hline
L'_{\theta_0} & L'_{\theta_{1s}} & L'_{\theta_{1c}} & L'_{\theta_{0T}} \\ \hline
0 & 0 & 0 & 0 \\ \hline
N'_{\theta_0} & N'_{\theta_{1s}} & N'_{\theta_{1c}} & N'_{\theta_{0T}}
\end{array}\right]
$$

The coefficients of the A matrix are called stability derivatives while that of the B matrix are called the control derivatives at trim. Eqn.(2.4) is the fundamental linearised form for describing the stability and response of small motion about a trim condition.

2.5 Handling qualities

Handling qualities denote the performance of the helicopter (ie, vehicle's response) to pilot's commands. The original definition of handling qualities by Cooper and Harper is [2]:

Those qualities or characteristics of an aircraft that governs the ease and precision with which a pilot is able to perform the tasks required in support of an aircraft role.

Goodness, or quality, according to Cooper-Harper, can be measured on a scale spanning three levels as shown in Fig.(2.2). Aircrafts are required to be Level 1 throughout the Operational Flight Envelope(OFE). Level 2 is acceptable in failed and emergency situations but Level 3 is considered as unacceptable.

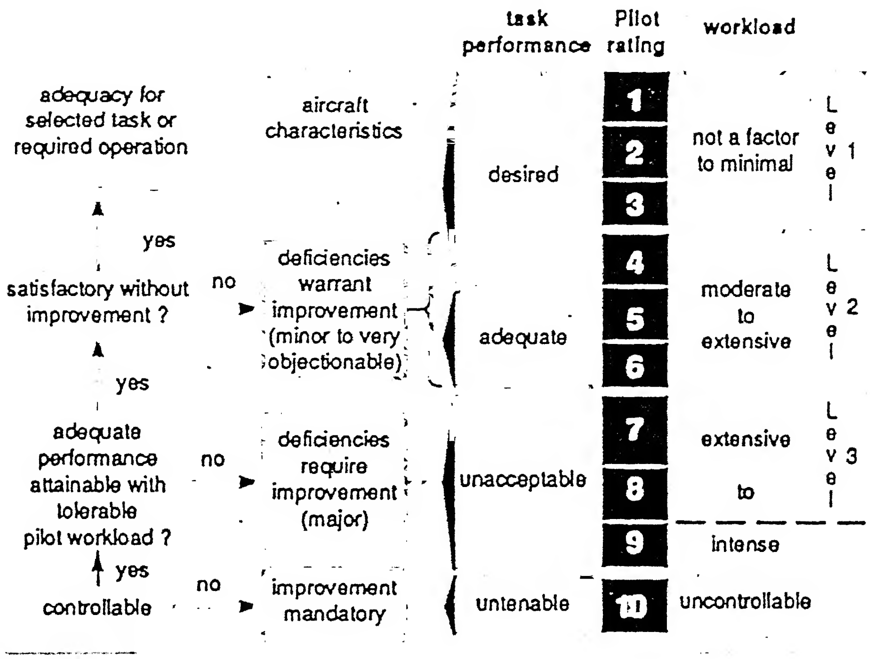

Figure 2.2: Cooper Harper Handling Qualities Rating Scale

The requirements of performing a given task drive Handling Qualities Requirements(HQRs and they are given in flying quality standards, like the US Army's Aeronautical Design Standard for handling qualities (ADS-33) and MIL-H-8501A. The criteria in these standards quantify responsiveness and sensitivity of the vehicle and lay down mandatory quality boundaries on measurable parameters, which are needed as design targets during design and certification stages.

2.5.1 Stability characteristics

Dynamical stability at trim refers to the ability of the vehicle to resist itself from diverging away from trim under small disturbances. The linearized perturbational equations are used to obtain the stability characteristics of the base helicopter. The longitudinal dynamics of the helicopter has an unstable phugoid (combination of pitch-heave-forward translation) which persists through out the speed range. The acceptability of these oscillations, according to the MIL-H-8501A stability requirements in the two flight regimes are given in Table 2.1.

Table 2.1: Summary of MIL-H-8501A Stability Requirements

Period	Damping Requirement	
	Visual Flight	Instrument Flight
$<$ 5 sec	1/2 amplitude in 2 cycles	1/2 amplitude in 1 cycle
5-10 sec	Atleast lightly damped	1/2 amplitude in 2 cycles
10-20 sec	Not double in 10 sec	Atleast lightly damped
$>$ 20 sec	No requirement.	Not double in 20 sec

2.5.2 Control characteristics

Control characteristics of helicopter deal with the problem of evaluating the response of the system to a given blade control input. Generally, the study involves analysing the response of the system to each of the four control inputs. The main rotor collective is not treated as a short term control. So the response to this input is not considered.

Pitch,Roll and Yaw response to control inputs

The total control response is contributed by (i)initial response and (ii)the steady state response. Generally for accurate manoeuvring. it is the initial response that plays a major role. The response behaviour is always analysed using the linearised perturbational equations developed earlier.

Control characteristics or handling quality requirements of a helicopter are specified in MIL-H-8501A. These requirements are shown in Fig.(2.3) below.

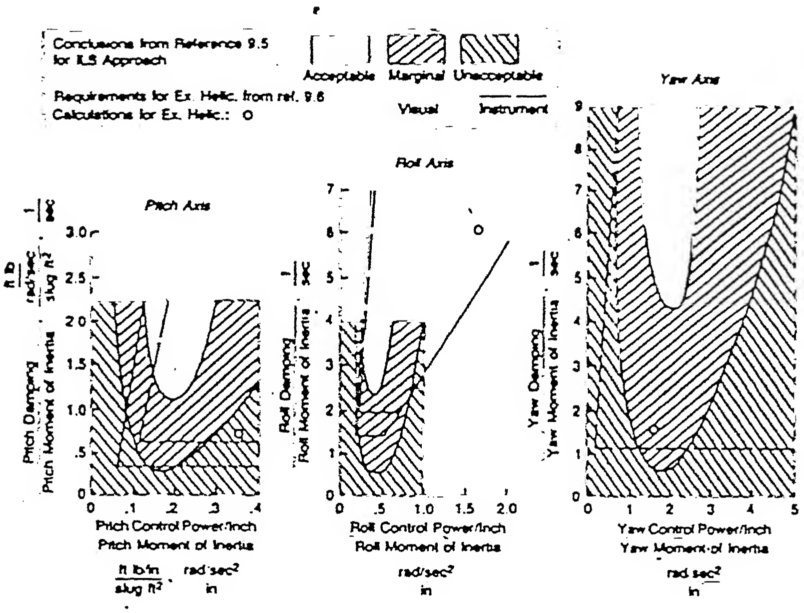

Figure 2.3: MIL-H-8501A Control Requirements

These three figures correspond to response in uncoupled pitch, roll and yaw manoeuvres. The limits specified are based on pilot's opinion. If the response of the vehicle for a unit step pilot input is very slow and small, then the pilot will find the vehicle to be sluggish and if the response is too large, then the vehicle is over sensitive. The parameters of the two axes correspond to the control power and control damping. These will be explained by following the uncoupled motion in any one axis are shown in Fig.(2.3). For example the uncoupled roll equation can be written as

$$\dot{p} = \{\frac{\partial L}{\partial p}p + \frac{\partial L}{\partial \theta_{1c}}\theta_{1c}\}/I_{xx} \tag{2.5}$$

$$\tag{2.6}$$

Note : $L_p = \frac{\partial L}{\partial p}/I_{xx}$; $L_{\theta_{1c}} = \frac{\partial L}{\partial \theta_{1c}}/I_{xx}$

$\rightarrow L_p = \bar{L_p}/I_{xx}$; $L_{\theta_{1c}} = \bar{L_{\theta_{1c}}}/I_{xx}$

Its solution is given by

$$p = \frac{L_{\theta_{1c}}/I_{xx}}{L_p/I_{xx}}[1 - \epsilon^{L_p/I_{xx}t}]\theta_{1c} \tag{2.7}$$

The handling quality requirements are based on the two parameters representing the initial angular acceleration of roll motion for unit control input and the damping in roll. viz

$L_{\theta_{1c}}$ Vs L_p

Similar parameters are used for pitch and yaw with the corresponding control moment, damping and inertia.

From the foregoing discussion, it is evident that the control system designer must consider the various coupling effects in the system dynamics and also any nonlinearities present in the elements. Due to complexity of the problem, the design of a robust control system for a rotorcraft, meeting the prescribed HQ performance specifications under anticipated disturbances and uncertainties is a challenging task.

Chapter 3

Design and Development of Flight Control Laws

3.1 Introduction

This chapter is devoted to the development of control laws for stability and control augmentation for a helicopter. Further the design of a robust stabilizing controller for the helicopter model at hover using the multiplicative description of plant uncertainty is also discussed. The problem of designing control laws for the AFCS requires modeling of 1)the vehicle dynamics in different trim conditions and understanding the extent of modelling uncertainties and 2) a quantitative list of specifications on performance. These were discussed in the last chapter.

3.2 Helicopter model description

For FCS design, it is necessary to consider the dynamics of other components such as the cyclic mixer, the actuator and the mechanical linkage between the pilot sticks and the actuator in addition to the vehicle dynamics. At a functional level the closed loop system formed by these subsystems is described schematically in Fig.(3.1). For each of the functional blocks shown above the dynamic equations are developed below. For the vehicle dynamics these are linearised equations at a specified trim condition.

1. Vehicle dynamics: A detailed description of the vehicle dynamics is given in chapter
2. For the Stability and Control Augmentation System(SCAS)design, only the cyclic

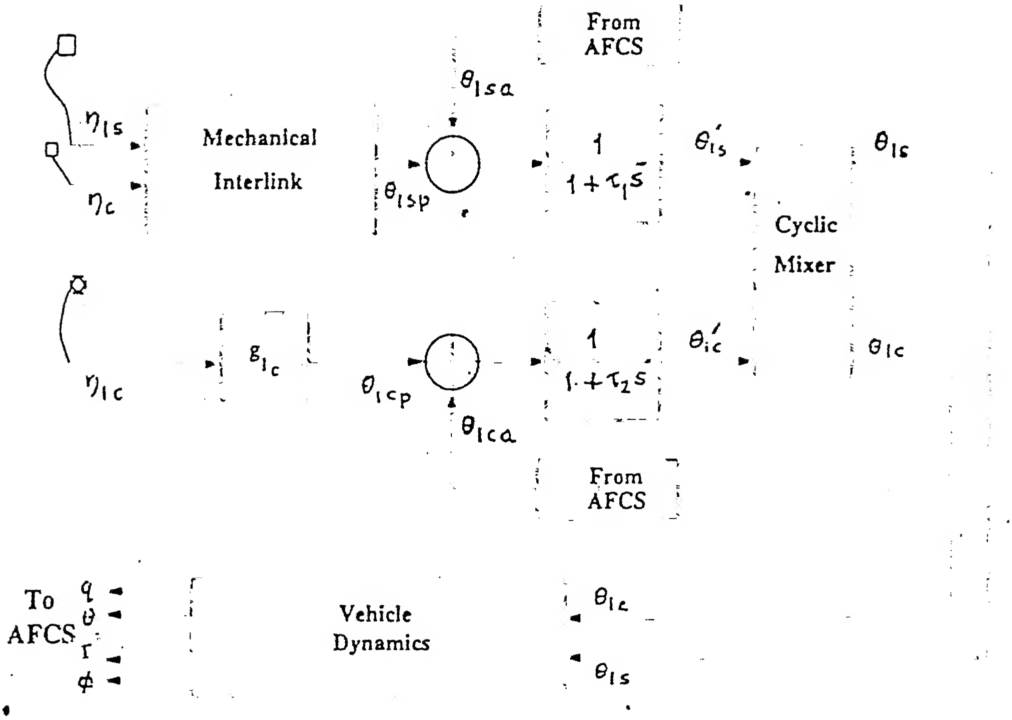

Figure 3.1: Vehicle and Components

inputs are considered. The equations of motion is given by

$$\dot{x} = Ax + Bu \tag{3.1}$$

where

$$x^T = [u,\ v,\ w,\ p,\ q,\ r,\ \theta,\ \phi]$$
$$u^T = [\theta_{1s},\ \theta_{1c}]$$

The decoupled longitudinal and lateral dynamics is used in Control Law(CLAW) design and is written as

$$\dot{x_{ln}} = A_{ln}x_{ln} + B_{ln}u_{ln}, \tag{3.2}$$

with

$$x_{ln}^T = [u,\ w,\ q,\ \theta]$$
$$u_{ln} = [\theta_{1c}]$$

for longitudinal axis and

$$\dot{x_{lt}} = A_{lt}x_{lt} + B_{lt}u_{lt}, \tag{3.3}$$

with

$$\begin{aligned} x_{lt}^T &= [v,\ p,\ r,\ \phi], \\ u_{lt} &= [\theta_{1c}] \end{aligned}$$

for lateral axis. The model of the helicopter at hover is given in Appendix D. From this data, the matrices in Eqns.(3.2) and (3.3) for the decopled longitudinal and lateral dynamics are as follows.

$$A_{ln} = \begin{bmatrix} -0.0157 & 0.0312 & 0.0053 & -0.1703 \\ 0.0048 & -0.3346 & 0.0045 & -0.0172 \\ 0.9390 & -0.0905 & -1.3220 & 0.0000 \\ 0.0000 & 0.0000 & 0.9900 & 0.0000 \end{bmatrix}$$

$$B_{ln} = \begin{bmatrix} -0.2061 \\ -0.0103 \\ 21.5493 \\ 0.0000 \end{bmatrix} \tag{3.4}$$

$$A_{lt} = \begin{bmatrix} -0.0001 & -0.0084 & 0.0090 & 0.1701 \\ -6.6900 & -5.1620 & -0.0426 & 0.0000 \\ 5.6710 & -0.5650 & -0.9319 & 0.0000 \\ 0.0000 & 1.0000 & 0.1012 & 0.0000 \end{bmatrix}$$

$$B_{lt} = \begin{bmatrix} 0.1950 \\ 3.0026 \\ 14.3031 \\ 0.0000 \end{bmatrix} \tag{3.5}$$

2. Cyclic mixer: For the cyclic mixer shown in Fig.(3.1) the equations are algebraic. ie,

$$\begin{bmatrix} \theta_{1s} \\ \theta_{1c} \end{bmatrix} = G \begin{bmatrix} \theta'_{1s} \\ \theta'_{1c} \end{bmatrix} \tag{3.6}$$

The effect of cyclic mixer has already been included in the matrices A and B, given for the particular model.

3. The actuators: The actuators are modelled as first order lags with time constants τ_1, τ_2 both between 25 and 100ms. Their equations are

$$\begin{bmatrix} \dot{\theta}'_{1s} \\ \dot{\theta}'_{1c} \end{bmatrix} = \begin{bmatrix} -1/\tau_1 & 0 \\ 0 & -1/\tau_2 \end{bmatrix} \begin{bmatrix} \theta'_{1s} \\ \theta'_{1c} \end{bmatrix} + \begin{bmatrix} 1/\tau_1 & 0 \\ 0 & 1/\tau_2 \end{bmatrix} \begin{bmatrix} \theta_{1sp} + \theta_{1sa} \\ \theta_{1cp} + \theta_{1ca} \end{bmatrix} \quad (3.7)$$

For the given problem , the value of both τ_1 and τ_2 have been taken as 25ms.

4. Pilot's inputs: The components of inputs θ_{1sp} and θ_{1cP} are obtained from the pilot's inputs through the mechanical linkage. The function of these linkages is to scale the pilot's inputs by constants. The particular A and B in Eqn.(3.2) includes the dynamics of the linkage also.

In order to design the control law, the above models have to be connected together to form an appropriate i/p-o/p system. This is done in the later sections.

3.3 MIL-H-8501 performance specification

The goal of any rotorcraft control system design is to achieve "Level 1" handling qualities performance. The MIL-H-8501A specifications for handling qualities is listed in chapter 2 and is reproduced below. As seen in chapter 2, the unstable phugoid persists throughout the flight envelope. The acceptability of oscillations in the two flight regimes are given in the Table 3.1.

Table 3.1: Summary of MIL-H-8501A Stability Requirements

Period	Damping Requirements	
	Visual Flight	Instrument Flight
< 5 sec	1/2 amplitude in 2 cycles	1/2 amplitude in 1 cycle
5-10 sec	Atleast lightly damped	1/2 amplitude in 2 cycles
10-20 sec	Not double in 10 sec	Atleast lightly damped
> 20 sec	No requirement	Not double in 20 sec

The control characteristics or handling quality requirements of a helicopter, as specified in MIL-H-8501A are shown in the Fig.(3.2)

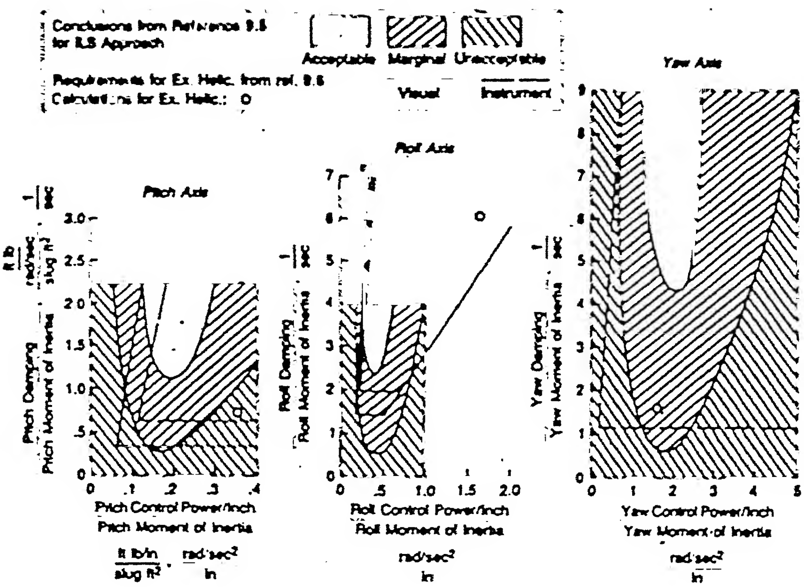

Figure 3.2: MIL-H-8501A Control Requirements

3.4 Approach to control law design

The control law design for stability and control augmentation uses the decoupled longitudinal and lateral dynamics of the helicopter and separate controllers are developed for each axis. The closed loop system including the controller should satisfy the performance requirements listed in Table 3.1 and Fig.(3.2). But the two types of performance requirements ie. the stability requirement and the handling requirement are interdependent. Hence the design procedure involves designing the closed loop law satisfying the stability requirements first, followed by the development of forward compensator to tailor the handling requirements [18].

3.4.1 Stabilization

The design of control laws for stabilization is first considered. For this purpose, augmented system model, which is the series combination of the vehicle dynamics and the actuator dynamics is required. The state vector, input vector and output vector for the augmented model in the longitudinal axis are chosen as,

$$x_1^T = [\, u,\ w,\ q,\ \theta,\ \theta'_{1c}]$$

$$
\begin{aligned}
u_1^T &= [\theta_{1ca}] \\
y_1^T &= [q, \theta]
\end{aligned}
$$

The state vector above consists of vector components x_{ln}, of the vehicle dynamics and x_a, those of the actuator dynamics. The dynamic model, obtained by connecting these components, using the data given, is

$$
\begin{aligned}
\dot{x} &= A_{aug}x_1 + B_{aug}u_1 \\
y &= C_{aug}x_1 + D_{aug}u_1
\end{aligned} \tag{3.8}
$$

with

$$
A_{aug} = \begin{bmatrix} -0.0157 & 0.0312 & 0.0053 & -0.1703 & -8.2440 \\ 0.0048 & -0.3346 & 0.0045 & -0.0172 & -0.4122 \\ 0.9390 & -0.0905 & -1.3220 & 0.0000 & 861.9700 \\ 0.0000 & 0.0000 & 0.9900 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & -40.0000 \end{bmatrix},
$$

$$
B_{aug} = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 1.0000 \end{bmatrix}
$$

$$
C_{aug} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
$$

$$
D_{aug} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

Stabilization problem: To find a controller with input y and output u such that the closed loop system has eigen values in the specified region so as to satisfy the stability requirements given in Table 3.1. Using the parametrization formula a family of controllers satisfying this criteria can be found(see Appendix A). One such controller is given below. The steps leading to this controller are as follows.

1. Fix a set of 5 distinct complex numbers so as to satisfy the criteria in Table 3.1.

2. Find matrices K and H such that A-BK and A-HC have eigen values at the above complex numbers.

3. The formula for the controller transfer matrix is then given by

$$C(s) = X(s)Y^{-1}(s) \tag{3.9}$$

where

$$X(s) = \begin{pmatrix} A - BK & H \\ -K & 0 \end{pmatrix}$$
$$Y(s) = \begin{pmatrix} A - BK & H \\ C - DK & I \end{pmatrix}$$

4. The stability margin,closed loop sensitivity function and closed loop eigen values are then determined.

In order to design the longitudinal controller, the complex numbers in step 1 are chosen as

$$p = \begin{bmatrix} -40 & -1.4050 & -0.3386 & -0.2 \pm 0.3331i \end{bmatrix}$$

The matrices K and H are obtained as

$$K = \begin{bmatrix} -0.0990 & -0.0058 & 0.0213 & 0.0429 & 0.4713 \end{bmatrix}$$
$$H = \begin{bmatrix} 0.3992 & -0.1703 \\ 0.0317 & -0.0172 \\ -0.9337 & 0.0000 \\ 0.9900 & 1.4050 \\ 0.0000 & 0.0000 \end{bmatrix}$$

The longitudinal controller at hover is then obtained as

$$C(s) = \begin{pmatrix} A_c & B_c \\ C_c & D_c \end{pmatrix} \tag{3.10}$$

where A_c, B_c, C_c, D_c are

$$A_c = \begin{bmatrix} -0.3636 & -0.4560 & 20.4644 & 34.4671 & 170.6367 \\ -0.1194 & -2.1508 & 74.5013 & 125.8861 & 621.7803 \\ 0.0000 & 0.8774 & -63.2956 & -105.5043 & -525.4513 \\ 0.0000 & 0.0000 & 14.8447 & 23.0912 & 118.2093 \\ 0.0000 & 0.0000 & 0.0000 & 0.2468 & 0.1039 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -0.1360 & 0.0902 \\ -0.0571 & 0.9339 \\ 1.8189 & 0.7776 \\ -0.8317 & -1.5719 \\ -0.0499 & 0.2257 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.3423 \end{bmatrix}$$

$$D_c = \begin{bmatrix} 0.0000 & 0.0000 \end{bmatrix}$$

Moving along the same lines, the controller in the lateral direction can be obtained for the given model at hover. It is given below. The complex numbers in step 1 are chosen as

$$P = \begin{bmatrix} -0.2 \pm 0.3331i & -5.2188 & -2.0000 & -40.0000 \end{bmatrix}$$

The lateral controller is

$$A_c = \begin{bmatrix} 0.3778 & 4.1278 & -181.0253 & 187.2714 & 473.1824 \\ -0.4111 & -4.5483 & 66.4548 & -58.9959 & -149.4250 \\ 0.0000 & 0.4072 & 7.4586 & -9.9965 & -23.8347 \\ 0.0000 & 0.0000 & 47.8798 & -50.8002 & -127.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.4727 & -2.0315 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 1.247 & 12.1144 \\ 0.9881 & -3.0068 \\ -0.3516 & -0.9236 \\ -0.8198 & -4.1664 \\ 0.1690 & 0.8303 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.3898 \end{bmatrix}$$

$$D_c = \begin{bmatrix} 0.0000 & 0.0000 \end{bmatrix}$$

The same procedure is carried out for the helicopter model at 200kph. The model is given in Appendix D. The longitudinal controller has been found out and is given below. The pole location chosen in the design criteria is

$$P = \begin{bmatrix} -0.2 \pm 0.3331i & 1.4050 & -0.3386 & -40.0000 \end{bmatrix}$$

The controller is

$$A_c = \begin{bmatrix} -0.2829 & -0.3710 & -26.5326 & 14.8712 & 446.5495 \\ 0.0891 & -0.4112 & 5.1943 & -1.6459 & -77.0964 \\ 0.0000 & 0.6779 & -27.7090 & 12.8070 & 432.6279 \\ 0.0000 & 0.0000 & 28.0092 & -14.7179 & -458.3225 \\ 0.0000 & 0.0000 & 0.0000 & 0.1179 & -1.5189 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 2.0253 & 3.7937 \\ 0.9013 & 0.5828 \\ 0.2947 & 0.8890 \\ -2.8761 & -3.1318 \\ 0.0984 & 0.1166 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.7695 \end{bmatrix}$$

$$D_c = \begin{bmatrix} 0.0000 & 0.0000 \end{bmatrix}$$

Lateral dynamics of the model at 200kph is stable. Its eigen. values are

$$\begin{bmatrix} -0.6894 \pm 2.3547i & -5.8189 & -0.0416 & -40.0000 \end{bmatrix}$$

So there is no need of designing the control law in the lateral direction.

Now the closed loop stabilizing control law for the whole coupled system at hover has been found out using the same design strategy. The controller has been found out and is given below. The pole location chosen in step 1 is

$$P = \begin{bmatrix} -40.0 & -40.0 & 0.5 & -2.0 & -0.5 \pm 1.27i & -0.3 \pm 0.4i & -0.2 \pm 0.35i \end{bmatrix}$$

The controller is $A_c=$

$$\begin{bmatrix} -0.27 & 1.07 & -19.30 & -48.22 & 195.90 & 169.25 & -84.25 & -67.44 & -503.75 & 587.69 \\ -0.01 & -0.47 & 17.44 & -84.25 & 328.24 & 365.19 & -35.13 & -92.78 & 380.24 & -184.72 \\ 0.01 & 0.07 & 0.59 & -8.18 & 17.41 & 17.30 & 1.54 & -5.44 & 1.05 & 2.15 \\ 0.01 & 0.08 & 0.72 & -6.65 & 10.26 & 9.28 & 3.51 & -2.07 & 2.17 & -0.97 \\ 0.00 & 0.00 & -0.52 & 4.43 & -22.49 & -24.78 & 5.26 & 7.32 & -3.42 & -11.02 \\ 0.00 & 0.00 & 0.68 & 3.39 & -11.17 & -18.01 & 1.07 & -6.93 & 2.49 & -8.35 \\ 0.00 & 0.00 & 0.00 & 0.00 & 5.57 & 4.48 & -2.97 & -5.61 & -1.10 & 1.56 \\ 0.00 & 0.00 & 0.00 & 0.00 & 3.42 & -7.30 & -4.01 & -37.14 & 0.74 & 2.64 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 1.64 & -2.4 & -0.81 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.12 & 0.16 & 2.39 & -0.42 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 44.71 & -7.22 & 2.32 & -88.26 \\ -34.81 & 3.15 & -2.46 & 15.51 \\ 0.30 & 0.31 & 0.86 & -0.93 \\ 0.32 & 2.07 & 0.89 & -1.00 \\ 0.27 & 0.59 & 0.53 & 0.76 \\ -0.54 & -16.84 & -0.30 & -1.51 \\ -0.08 & -6.09 & -0.35 & -0.70 \\ -1.29 & -53.57 & 0.10 & 0.49 \\ 0.01 & 2.25 & -0.12 & -0.58 \\ 0.00 & 0.24 & -0.01 & 0.03 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.90 & 0.30 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.10 & 8.30 \end{bmatrix} \quad D_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3.4.2 Handling qualities criterion

The next step is to design a forward compensator to achieve the desired response qualities. For this the closed loop equations have to be formed. The transfer function of the closed loop system with θ_{1c} as the input and y as output is given by

$$T_{yp} = \begin{pmatrix} A_{cl} & B_{cl} \\ C_{cl} & 0 \end{pmatrix} \tag{3.11}$$

where

$$
\begin{aligned}
A_{cl} &= \begin{bmatrix} A_{aug} + B_{aug} D_c C_{aug} & B_{aug} C_c \\ B_c C_{aug} & A_c \end{bmatrix} \\
B_{cl} &= \begin{bmatrix} B_{aug} \\ 0 \end{bmatrix} \\
C_{cl} &= \begin{bmatrix} C_{aug} & 0 \end{bmatrix}
\end{aligned}
$$

In order to satisfy the specifications on handling qualities, a forward compensator is designed so as to provide additional i/p summed up at the acuator i/p. The gain of this forward compensator is then chosen to satisfy the HQRs of Fig.3.2. The relevant equations are

1. Closed loop equations :

$$
\begin{aligned}
\dot{x}_{cl} &= A_{cl} x_{cl} + B_{cl} v_p \\
y &= C_{cl} x_{cl}
\end{aligned}
$$

2. Mechanical mixer :

$$
\theta_{1p} = K_p \eta
$$

where η is the pilot input.

3. Forward compensation block :

$$
x = F\eta
$$

4. Summation of inputs :

$$
v_p = (K_p + F)\eta
$$

5. Compensated system :

$$
\begin{aligned}
\dot{x}_{cl} &= A_{cl} x_{cl} + B_{cl}(K_p + F)\eta \\
y &= C_{cl} x_{cl}
\end{aligned}
$$

The above formula of the dynamical system from pilot input η to the output y then involves the gain block F which can be adjusted, thus meeting the response requirements without affecting the stability requirements met by the closed loop controller. The uncoupled equation for pitch in the closed loop system formed, for the problem at hover can be written as

$$\dot{q} = -1.322q + 0u$$

Now, a scalar f_1 is chosen as $f_1 = 0.2$ such that

$$\dot{q} = -1.322q + f_1 u \tag{3.12}$$

belongs to the acceptable region of Fig.(3.2). Similarly for the model at 200kph, the equation for pitch is

$$\dot{q} = -1.7540q + 0u$$

for which the scalar f_1 chosen is $f_1 = 0.2$ inorder for the control requirements to be met. Similarly the uncoupled equation for p and r at hover can be written as

$$\dot{p} = -5.1620p + 0u$$
$$\dot{r} = -0.9319r + 0u$$

As in the longitudinal case, we have to find out scalars f_2 and f_3 such that

$$\dot{p} = -5.1620p + f_2 u$$
$$\dot{r} = -0.9319r + f_3 u$$

meet the control specifications. For this f_3 is chosen as $f_3 = 2$, but f_2 cannot be found out, since the scale given in Fig.(3.2) for p is not sufficient. For the model at 200kph the equations for p and r are

$$\dot{p} = -5.592p + 0u$$
$$\dot{r} = -1.416r + 0u$$

Here also the scalar f_2 can not be found out since the scale given is not sufficient. f_3 has been found out to be $f_3 = 2$, for the control specifications to be met for the model at 200kph.

3.5 Design of control laws using PI techniques

Let us try to analyse the problem of closed loop stabilization using a PI controller. The output signal produced by the PI controller consists of two terms, one is proportional to the input signal and the other proportional to its integral. For longitudinal hover the output signal of the controller is

$$C(s) = K_1 \int \begin{bmatrix} q \\ \theta \end{bmatrix} dt + K_2 \begin{bmatrix} q \\ \theta \end{bmatrix}$$

K_1 and K_2 can be calculated so that the resulting closed loop system meets the MIL-H-8501 stability requirements. The values of K_1 and K_2 that makes the closed loop system stable are given in the Table 3.3.

Table 3.2: Values of K_1 and K_2 for the PI Controller

K_1	K_2
0.11	0.33
0.33	1.00
0.56	0.56
0.78	0.1
1.00	0.78
1.00	1.00

But in PI methods the compensator for arbitrary pole locations may not exist, unlike the case for state feedback compensator. Also thePI compensator may not give satisfactory response for the whole coupled system.

3.6 Robustness

No nominal plant model can emulate a physial plant perfectly. Thus in the design of control system, we are faced with the uncertainty of the model of the plant, originated

from various sources such as the neglected nonlinearity of the plant dynamics, the simplification of the plant model for the purpose of controller design, the variation of plant parameters during the operation and so on. For instance, the major uncertainties associated with a rotorcraft model are due to neglected rotor dynamics and unmodelled rate limit nonlinearities. Also, the aerodynamic coefficient vary with flight environment. However, the controller (designed for the simplified linear model) should maintain closed loop stability for the actual system in the face of such perturbations. This is the main concern of the robust stability problem. As shall be seen in the subsequent sections, this problem is elegantly addressed in the H_∞ framework.

3.6.1 Robust stabilization as a standard H_∞ optimization problem

In this section we pose the problem of robust stabilization as a standard 4-block problem. It is well known that, by making use of the stable coprime factorization theory [14], the problem of robust stabilization can be formulated as an H_∞ model matching problem. In this problem a stable transfer matrix R is determined such that the error $\| T_1 - T_2RT_3 \|_\infty < 1$ where T_1, T_2, T_3 are transfer matrices obtained from the plant and the uncertainty profile. However an algorithmic solution of the model matching problem has many drawbacks, particularly for the multivariable case. For this reason, it is desirable if a model matching problem can be formulated as a standard 4-block problem. This is carried out below.

Robust stabilization at hover :-

First we would like to determine whether a robust stabilizing controller can be developed at hover such that this controller also stabilizes the model at full speed. This analysis is carried out using the multiplicative description of uncertainty. Let P_0 denotes the transfer matrix of the dynamics that corresponds to hover. It is given by

$$P_0(j\omega) = \begin{bmatrix} P_1(j\omega) \\ P_2(j\omega) \end{bmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{3.13}$$

Recall that this is the model obtained in Eqn.(3.4). Let $\Delta(s)$ denotes any stable transfer matrix denoting a ball of uncertainty such that $\| \Delta \|_\infty < 1$. Then an uncertain set of

plants which includes P_0, is represented by the set [9, 16]

$$M(P_0, W) = \{\tilde{P} \mid \tilde{P} = (I + \Delta W)P_0, \ \| \Delta \|_\infty < 1\} \tag{3.14}$$

This is called multiplicative uncertainty around the nominal plant model P_0. Here W is chosen such that M covers the required set of models of the uncertain plant. For the present purpose we need to find a transfer function weight W such that if P_0 denotes the model at hover, then the model at full forward speed is also included in the set M. Frequency response design of W is described next. From Eqn.(3.14), it follows that

$$\| \tilde{P} - P_0 \| (\omega) \leq \| WP_0 \| (\omega) \ \forall \omega \in [0, \infty) \tag{3.15}$$

The full order model at 330kph is given in Appendix D. From this data, the decoupled longitudinal model $\tilde{P}$ is written as

$$\tilde{P}(j\omega) = \begin{bmatrix} \tilde{P}_1(j\omega) \\ \tilde{P}_2(j\omega) \end{bmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{3.16}$$

W is chosen to be of the simple diagonal form

$$W = \begin{bmatrix} a_1(s) & 0 \\ 0 & a_2(s) \end{bmatrix}$$

where $a_1(s)$ and $a_2(s)$ are selected such that Eqn.(3.15) is satisfied. ie,

$$| \tilde{P}_1(j\omega - P_1(j\omega) |^2 + | \tilde{P}_2(j\omega) - P_2(j\omega) |^2 \leq | a_1(j\omega)P_1(j\omega) |^2 + | a_2(j\omega)P_2(j\omega) |^2$$

Using the given data in Eqn.(3.13) and (3.16), $a_1(s)$ and $a_2(s)$ are found out to be

$$a_1(s) = \frac{(10s + 1)(0.1s + 1)}{4s^2 + 2.8s + 1}$$
$$a_2(s) = \frac{2(0.1s + 1)}{2s + 1}$$

Robust Stability Condition

The robust stabilizability condition for the above multiplicative description of the uncertainty around a nominal plant model P_0 is given by the following theorm [14].

Theorm:–

Let $M(P_0, W)$ and $W(s)$ are specified and suppose C stabilizes P_0. Then C stabilizes all P in the class $M(P_0, W)$ if and only if

$$\| [WP_0C(I + P_0C)^{-1}](j\omega) \|_\infty \leq 1 \ \forall \omega \in R \tag{3.17}$$

The robust stability condition of Eqn.(3.17) can be formulated as a 4-block problem as given below. The standard set up is shown in Fig.(3.3) for which the transfer matrix from w to z is [see Appendix B]

$$T_{zw} = G_{11} - G_{12}(I + G_{22}C)^{-1}G_{21} \tag{3.18}$$

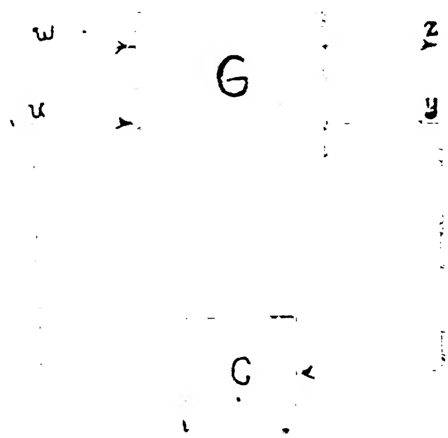

Figure 3.3: The Standard Block Diagram

We can convert the robust stability condition to the set-up of the standard problem by defining G so that in Fig.(3.3), the transfer matrix from w to z equals $WP_0C(I + P_0)^{-1}$. This is accomplished by taking $G_{11} = 0$, $G_{12} = -WP_0$, $G_{21} = I$, $G_{22} = P_0$. Hence the 4-block plant corresponding to the robust stability condition is

$$G = \begin{bmatrix} 0 & -WP_0 \\ I & P_0 \end{bmatrix} \tag{3.19}$$

with the state space model

$$G = \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix}$$

While checking the solvability of the standard 4-block problem using model matching theory[see Appendix C], it was found that $\| T_1(z) \| = 2.0069$, which should be less than 1 inorder for the robust stability problem to be solvable. Thus it turned out that the robust

stabilization of the whole flight envelope by including the two extreme flight configurations as modelled by the above W is not possible. Since there exist many other possible functions W which meet the requirement of including $\tilde{P}$ in M in Eqn.(3.14), the above failure to stabilize both hover and full forward speed is not conclusive. However it can be said that this would involve very complex controllers and hence may be impractical.

Now the robust stabilization at hover is carried out with a different W. ie,

$$W(s) = \begin{bmatrix} a_1'(s) & 0 \\ 0 & a_2'(s) \end{bmatrix}$$

where $a_1'(s)$ and $a_2'(s)$ are selected such that there is not much deviation between P and $\tilde{P}$ at low frequencies and at high frequencies there is large deviation. This description of unceraintty is infact more realistic since for most dynamic systems the model uncertainty begin from the mid frequency range while at very low frequencies the models are good approximation to the plant. For the present problem, $a_1'(s)$ and $a_2'(s)$ are chosen as

$$a_1'(s) = \frac{\tau_1 s + k_1}{0.1s + 1}$$

$$a_2'(s) = \frac{\tau_2 s + k_2}{0.1s + 1}$$

The solvability of the robust stabilizability condition is checked using the model matching theory. For different τ_1 and τ_2, the value of $\| T_1(z) \|$ is given in Table 3.2. In order for the 4-block problem to be solvable, $\| T_1(z) \| \leq 1$. So $\tau_1 = 0.3,\ k_1 = 0.1,\ \tau_2 = 0.3,\ k_2 = 0.1$ is taken for $a_1'(s)$ and $a_2'(s)$.

3.6.2 State space procedure

We now use the iterative state space algorithm of Glover and Doyle [8], to find the solution to the above problem of making $\| T_{zw} \| \leq 1$. This requires the condition (1), (2), (3), (4) to be fullfilled. See Appendix B. Hence these are first vertified for the above data.

For the particular W calculated above and for P_0 of Eqn.(3.13), the state space representation corresponding to Eqn.(3.19) is written as

Table 3.3: Variation of $\| T_1(z) \|$ with τ and k

τ_1	k_1	τ_2	k_2	$\| T_1(z) \|$
0.02	0.01	0.02	0.01	0.0386
0.2	0.1	0.02	0.01	0.1290
0.2	0.1	0.2	0.1	0.3860
0.3	0.1	0.2	0.1	0.3938
0.3	0.1	0.3	0.1	0.4563
0.03	0.01	0.3	0.1	0.4324
0.03	0.01	0.03	0.01	0.0456
0.3	0.1	0.03	0.01	0.1525

A=

$$\begin{bmatrix} -40.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -8.24 & -0.02 & 0.03 & 0.01 & -0.17 & 0.00 & 0.00 \\ -0.41 & 0.01 & -0.33 & 0.00 & -0.02 & 0.00 & 0.00 \\ 861.97 & 0.94 & -0.09 & -1.32 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.99 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & -10.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & -10.00 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad B_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & -3 & 0 & 29 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 29 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad D_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad D_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D_{22} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For the given data rank $D_{12} = 0$. Hence condition (2) is not satisfied. This is due to the fact that the control effect is not included in the performance specification(ie, robust

stability condition). Hence instead of this performance specification, we use the following regularised specification.

$$\left\| \begin{array}{c} WP_0C(I+P_0C)^{-1} \\ W_1C(I+P_0C)^{-1} \end{array} \right\|_\infty \leq 1 \tag{3.20}$$

clearly for $W_1 = \epsilon I$ and small ϵ, if a controller satisfies Eqn(3.20), then it also satisfies Eqn.(3.17). The effect of incorporating W_1 in the performance specification modifies the D_{12} matrix. Hence condition (2)is now satisfied. Even when ϵ is not small, the controller obtained for the modified specification of Eqn.(3.20) is still a robust stabilizing controller(which is more conservative). With the performance specification as in Eqn.(3.20), the 4-block plant is now modified as

$$G = \begin{bmatrix} 0 & \begin{bmatrix} WP_0 \\ W_1 \end{bmatrix} \\ I & P_0 \end{bmatrix} \tag{3.21}$$

Next, condition (3) and (4) are checked for the given data. The first system of condition (3) does not have any transmission zeros. For the second system no transmission zero is found to be lying on the imaginary axis. Since the transmission zeroes, if they exist are at the eigen values of A, a better numerical procedure to check condition(3) and (4) is to calculate the condition numbers of these matrices at the eigen values of A matrix. Thus it was verified that the given data satisfies both condition (3) and (4). Once these assumptions on the generalised plant are satisfied, we next check the existence conditions of the theorm given in Appendix B. Using the state space algorithm, the suboptimal controller for this problem is given by

$$K_{sub}(s) = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} \tag{3.22}$$

with

$$\hat{A} = \begin{bmatrix} -40.2498 & 0.0055 & 0.0385 & 42.8005 & 7.0336 & 358.0952 & -67.8392 \\ -8.2440 & -0.01577 & 0.0312 & -3.2054 & -3.2860 & 30.9651 & 29.9435 \\ -0.4122 & 0.0048 & -0.3346 & 36.9128 & 48.7944 & -356.2358 & -469.3768 \\ 861.9700 & 0.9390 & -0.0905 & 0.3285 & 0.3571 & -15.8961 & -3.4087 \\ 0.0000 & 0.0000 & 0.0000 & 1.4783 & 0.7750 & -4.7164 & -7.4548 \\ 0.0000 & 0.0000 & 0.0000 & 0.3725 & 0.5790 & -3.9325 & -5.5642 \\ 0.0000 & 0.0000 & 0.0000 & -0.0195 & 1.0569 & 0.1889 & -10.5477 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} -0.1360 & -0.0960 \\ 0.0074 & 0.0181 \\ -0.0563 & -0.2554 \\ -0.0061 & -0.0045 \\ -0.0004 & -0.0038 \\ -0.0002 & -0.0034 \\ 0.0000 & -0.0002 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} -0.24980.0055 & 0.0385 & 5.6181 & -0.0884 & 0.0198 & 0.0792 \end{bmatrix}$$

$$\hat{D} = \begin{bmatrix} 0.0000 & 0.0000 \end{bmatrix}$$

Thus the robust stabilizing controller has been formulated as a standard H_∞ optimization problem and a stabilizing controller has been found.

3.7 Summary of control laws designed

1. Flight condition : Hover

- Stability augmentation closed loop law:

$$y_{c_1} = C_1 \begin{bmatrix} q \\ \theta \end{bmatrix}$$

for the longitudinal axis with

$$C_1 = \begin{pmatrix} A_{c_1} & B_{c_1} \\ C_{c_1} & D_{c_1} \end{pmatrix}$$

where

$$A_{c_1} = \begin{bmatrix} -0.3636 & -0.4560 & 20.4644 & 34.4671 & 170.6367 \\ -0.1194 & -2.1508 & 74.5013 & 125.8861 & 621.7803 \\ 0.0000 & 0.8774 & -63.2956 & -105.5043 & -525.4513 \\ 0.0000 & 0.0000 & 14.8447 & 23.0912 & 118.2093 \\ 0.0000 & 0.0000 & 0.0000 & 0.2468 & 0.1039 \end{bmatrix}$$

$$B_{c_1} = \begin{bmatrix} -0.1360 & 0.0902 \\ -0.0571 & 0.9339 \\ 1.8189 & 0.7776 \\ -0.8317 & -1.5719 \\ -0.0499 & 0.2257 \end{bmatrix}$$

$$C_{c_1} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.3423 \end{bmatrix}$$

$$D_{c_1} = \begin{bmatrix} 0.0000 & 0.0000 \end{bmatrix}$$

and for the lateral axis

$$y_{c_2} = C_2 \begin{bmatrix} r \\ \phi \end{bmatrix}$$

$$C_2 = \begin{pmatrix} A_{c_2} & B_{c_2} \\ C_{c_2} & D_{c_2} \end{pmatrix}$$

where

$$A_{c_2} = \begin{bmatrix} 0.3778 & 4.1278 & -181.0253 & 187.2714 & 473.1824 \\ -0.4111 & -4.5483 & 66.4548 & -58.9959 & -149.4250 \\ 0.0000 & 0.4072 & 7.4586 & -9.9965 & -23.8347 \\ 0.0000 & 0.0000 & 47.8798 & -50.8002 & -127.0600 \\ 0.0000 & 0.0000 & 0.0000 & 0.4727 & -2.0315 \end{bmatrix}$$

$$B_{c_2} = \begin{bmatrix} 1.247 & 12.1144 \\ 0.9881 & -3.0068 \\ -0.3516 & -0.9236 \\ -0.8198 & -4.1664 \\ 0.1690 & 0.8303 \end{bmatrix}$$

$$C_{c_2} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.3898 \end{bmatrix}$$

$$D_{c_2} = \begin{bmatrix} 0.0000 & 0.0000 \end{bmatrix}$$

- Control augmentation compensator : Forward loop controller

$$y_{f_1} = C_{f_1} u_{f_1}$$

$C_{f_1} = 0.2$ in the longitudinal axis and for the lateral case it can not be found out as the scale given in the MIL specification is not sufficient.

2. Flight condition : Forward speed of 200kph

- Stability augmentation closed loop control law :

As in the above case of hover q and θ are used in feedback. The controller is

$$C_1 = \begin{pmatrix} A_{c_1} & B_{c_1} \\ C_{c_1} & D_{c_1} \end{pmatrix}$$

where

$$A_{c_1} = \begin{bmatrix} -0.2829 & -0.3710 & -26.5326 & 14.8712 & 446.5495 \\ 0.0891 & -0.4112 & 5.1943 & -1.6459 & -77.0964 \\ 0.0000 & 0.6779 & -27.7090 & 12.8070 & 432.6279 \\ 0.0000 & 0.0000 & 28.0092 & -14.7179 & -458.3225 \\ 0.0000 & 0.0000 & 0.0000 & 0.1179 & -1.5189 \end{bmatrix}$$

$$B_{c_1} = \begin{bmatrix} 2.0253 & 3.7937 \\ 0.9013 & 0.5828 \\ 0.2947 & 0.8890 \\ -2.8761 & -3.1318 \\ 0.0984 & 0.1166 \end{bmatrix}$$

$$
\begin{aligned}
C_{c_1} &= \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.7695 \end{bmatrix} \\
D_{c_1} &= \begin{bmatrix} 0.0000 & 0.0000 \end{bmatrix}
\end{aligned}
$$

The lateral dynamics is stable. So there is no need of designing the controller.

- Control augmentation compensator :Forward loop controller

  $C_{f_1} = 0.2$ for the longitudinal model and again for the lateral model the scale is insufficient.

3. Robust stabilization :

- Nominal condition : Hover

  Multiplicative uncertainty $M(P_0, W)$ is used where

$$
W = \begin{bmatrix} \frac{(10s+1)(0.1s+1)}{(1s^2+2.8s+1)} & 0 \\ 0 & \frac{(0.2s+2)}{(2s+1)} \end{bmatrix}
$$

  $M(P_0, W)$ includes the full forward speed. Controller does not exist in this case.

- Nominal condition : Hover

$$
\begin{aligned}
W &= \begin{bmatrix} \frac{(0.3s+0.1)}{(0.1s+1)} & 0 \\ 0 & \frac{(0.3s+0.1)}{(0.1s+1)} \end{bmatrix} \\
Y_c &= C u_c \\
C &= \left(\begin{array}{cc} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{array} \right]
\end{aligned}
$$

$\hat{A}$, $\hat{B}$, $\hat{C}$ are

$$
\hat{A} = \begin{bmatrix}
-40.2498 & 0.0055 & 0.0385 & 42.8005 & 7.0336 & 358.0952 & -67.8392 \\
-8.2440 & -0.01577 & 0.0312 & -3.2054 & -3.2860 & 30.9651 & 29.9435 \\
-0.4122 & 0.0048 & -0.3346 & 36.9128 & 48.7944 & -356.2358 & -169.376 \\
861.9700 & 0.9390 & -0.0905 & 0.3285 & 0.3571 & -15.8961 & -3.4087 \\
0.0000 & 0.0000 & 0.0000 & 1.4783 & 0.7750 & -4.7164 & -7.4548 \\
0.0000 & 0.0000 & 0.0000 & 0.3725 & 0.5790 & -3.9325 & -5.5642 \\
0.0000 & 0.0000 & 0.0000 & -0.0195 & 1.0569 & 0.1889 & -10.5477
\end{bmatrix}
$$

$$\hat{B} = \begin{bmatrix} -0.1360 & -0.0960 \\ 0.0074 & 0.0181 \\ -0.0563 & -0.2554 \\ -0.0061 & -0.0045 \\ -0.0004 & -0.0038 \\ -0.0002 & -0.0034 \\ 0.0000 & -0.0002 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} -0.2498 & 0.0055 & 0.0385 & 5.6181 & -0.0884 & 0.0198 & 0.0792 \end{bmatrix}$$

$$\hat{D} = \begin{bmatrix} 0.0000 & 0.0000 \end{bmatrix}.$$

Chapter 4

Results And Discussion

Several simulation studies were performed to analyse the dynamic performance of the closed loop system for controllers designed in the last chapter. The results of these studies are presented in three sections. The first and second sections describe the simulation results of the decoupled control laws developed at hover and at a forward speed of 200kph. In the last section, the results pertaining to the simulation of the closed loop full order system at hover is presented.

4.1 Simulation results of the decoupled control laws at hover

The mathematical model of the helicopter at hover is given in Appendix D. The system dynamics is decoupled into the longitudinal and lateral axis components and two separate controllers are designed for each axis as described in section 3.4.

Longitudinal dynamics

The longitudinal dynamic model of the helicopter at hover is unstable and the control law of Eqn.(3.10) makes the closed loop system stable. The poles of the uncompensated and compensated longitudinal model is given in Table 4.1. From Table 4.1, it can be seen that the uncompensated longitudinal model has one unstable complex conjugate pole. This pole can be relocated in the left half s-plane using pole placement techniques. In the compensated the poles have been relocated at -0.2±0.3331i. The sensitivity function S, of the compensated system is found out and $\| S \|_\infty$ has been calculated as 9.5640. The high value of $\| S \|_\infty$ indicates that the system is highly sensitive. The reason for

Table 4.1: Poles-longitudinal dynamics

uncompensated system	compensated system
-1.4050	-1.4050(2)
$0.0356 \pm 0.3331i$	$-0.2 \pm 0.3331i(2)$
-0.3386	-0.3386(2)
-40.0000	-40.0000(2)

this high sensitivity may be due to the fact that sensitivity minimization has not been considered in the design criteria. The response of the closed loop system for unit step input in longitudinal cyclic(θ_{1s}) are shown in Fig.(4.1)-(4.4) for all the state variables. For comparison, the step response is evaluated for three different pole locations. From Fig.(4.1) it can be seen that for a longitudinal input, the forward velocity (u_h) of the vehicle will change to a new value. However it must be noted that the response curves are not valid beyond initial few(2-3) seconds. Because, once the vehicle starts moving forward the system transfer function will change.

The step response of the vertical velocity component w_h is shown in Fig.(4.2). The change in w_h is very small. So it can be said that the influence of θ_{1s} on w_h is minimal. The response of pitch rate q to longitudinal cyclic input θ_{1s} of unit magnitude is shown in Fig.(4.3). The response of pitch angle θ to a unit step longitudinal cyclic input is shown in Fig.(4.4). Comparing the figures, it is observed that for the complex conjugate pole at $-0.2 \pm 0.3331i$, the initial response is more sensitive in comparison to the other two pole locations($-0.4 \pm 0.3331i$ and $-0.7 \pm 0.3331i$).

Lateral dynamics

The lateral dynamics of the helicopter at hover is unstable. So a controller has been designed to make it stable. The poles of the uncompensated and compensated system are shown in Table 4.2. The response of the compensated system for a unit lateral cyclic step(θ_{1c}) are shown in Figs.(4.5)-(4.8). Each of the figures shows the response of all the states, for three different pole locations($-0.2 \pm 0.3331i, -0.4 \pm 0.3331i, -0.7 \pm 0.3331i$). The results shown in Figs.(4.5)-(4.8) indicates that for the pole at $-0.2 \pm 0.3331i$ is more sensitive than at other locations.

Table 4.2: Poles-lateral dynamics

uncompensated system	compensated system
-5.2188	-5.2188(2)
-0.8794	-2.0000 (2)
$0.0021 \pm 0.3407i$	$-0.2 \pm 0.3331i(2)$
-40.0000	-40.0000(2)

4.2 Simulation results of the decoupled control laws at 200kph

A controller for stability is designed for helicopter operating at a forward speed of 200kph. The poles of the uncompensated and compensated longitudinal dynamics is given in Table 4.3. The response of the compensated system to a unit step input in longitudinal cyclic

Table 4.3: Poles- longitudinal dynamics at 200kph

uncompensated system	compensated system
-40.0000	-40.0000(2)
$-0.4595 \pm 0.3579i$	-1.4050(2)
$0.6359 \pm 0.4262i$	-0.3386(2)
	$-0.2 \pm 0.3331i(2)$

(θ_{1s}) is shown in Fig.(4.9) for all the variables. The characteristics of the initial response is similar to that observed in hover. The lateral dynamics of the model at 200kph is found to be stable. The poles of the lateral model are $-0.6894 \pm 2.3547i$, -5.8189, -0.0416, -40. So there is no need of designing the controller for stability augmentation.

4.3 Simulation of the controller for the coupled system at hover

The coupled system behaviour is investigated using the decoupled longitudinal and lateral controllers. The poles of the coupled system both uncompensated and compensated are given in Table 4.4.

Table 4.4: Poles-coupled system

uncompensated system	compensated system
-40(2)	-39.9937(2)
-5.1082	-0.3348
-1.3138	1.0781
$-0.7953 \pm 1.2689i$	$-40.0039 \pm 0.0097i$
$0.1375 \pm 0.3462i$	$0.1744 \pm 0.5198i$
$-0.0143 \pm 0.3609i$	$-5.1944 \pm 0.1040i$
	$-2.2096 \pm 0.4830i$
	$-1.1687 \pm 1.5491i$
	$-0.3405 \pm 0.6973i$
	$-0.33410.4429i$
	$-0.1827 \pm 0.1150i$

It can be seen from Table 4.4 that the uncompensated system is unstable. It does not satisfy MIL-H-8501A stability requirements because the frequency of oscillation of the unstable mode is 18.14 seconds. Its real part is such that the magnitude of the signal doubles in every 5.04 seconds. The compensated system with decoupled longitudinal and lateral controllers(designed for pole placement at $-0.2 \pm 0.3331i$) is also unstable. In addition it does not meet the MIL specification on stability requirements, since the frequency of oscillation of unstable mode is 12.57 seconds with the real part such that its magnitude doubles in every 4.23 seconds.

The response of the system was calculated for two different controllers designed for placement of poles at either $-0.2 \pm 0.3331i$ or $-0.4 \pm 0.3331i$. It can be seen from the

figures that for both set of controllers the coupled system shows unstable behaviour. The system with the controller designed for poles at $-0.4 \pm 0.3331i$ seems to be more unstable compared to the controller designed for poles at $-0.2 \pm 0.3331i$.

Since the compensated system with the decoupled controllers is unstable, a new controller is designed for the full order system. The poles of the uncompensated and compensated system are shown in Table 4.5. Thus it can be seen from the Table that the compensated system is stable.

Table 4.5: Poles-coupled system

-5.1082	-5.0000
-1.3138	-2.0000
$-0.7953 \pm 1.2689i$	$-0.5 \pm 1.2689i$
$-0.0143 \pm 0.3609i$	$-0.3 \pm 0.4i(2)$
$0.1375 \pm 0.3462i$	$-0.2 \pm 0.3462i(2)$
-40.0000(2)	-40.0000(4)

The step response of the compensated system are shown in Fig.(4.18) and (4.19) for longitudinal and lateral state variables. From these figures it can be seen that when both longitudinal and lateral cyclic step are appied to the compensated full order model, the longitudinal and lateral variables behave in a way similar to the decoupled longitudinal and lateral model.

Based on the limited results presented above, one can conclude that care must be exercised in using the controller designed for the decoupled dynamics for the full order system. Hence it may be beneficial to design for the full order system.

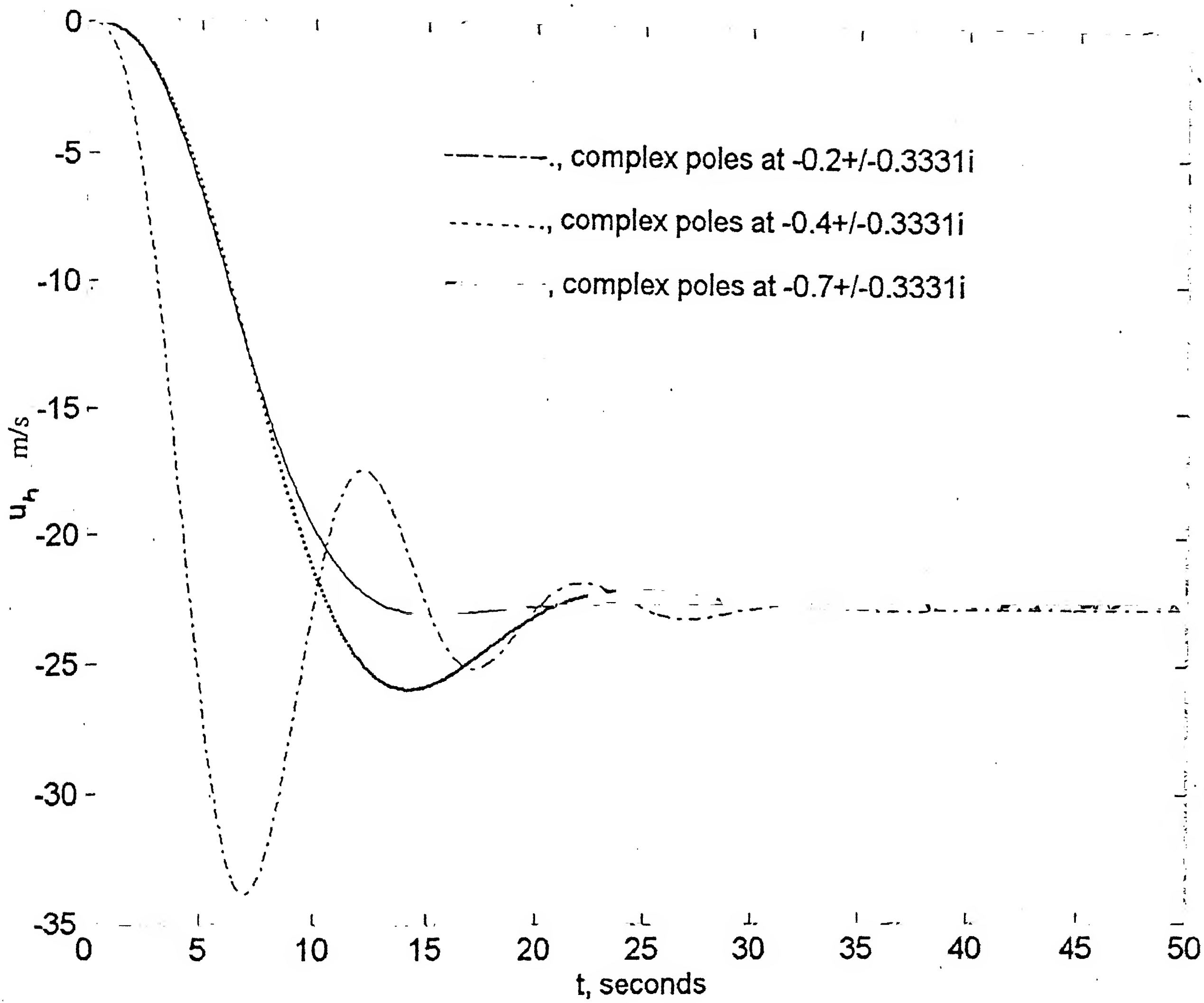

Figure 4.1: Response of u_h to a step i/p in θ_{1s}-Hover

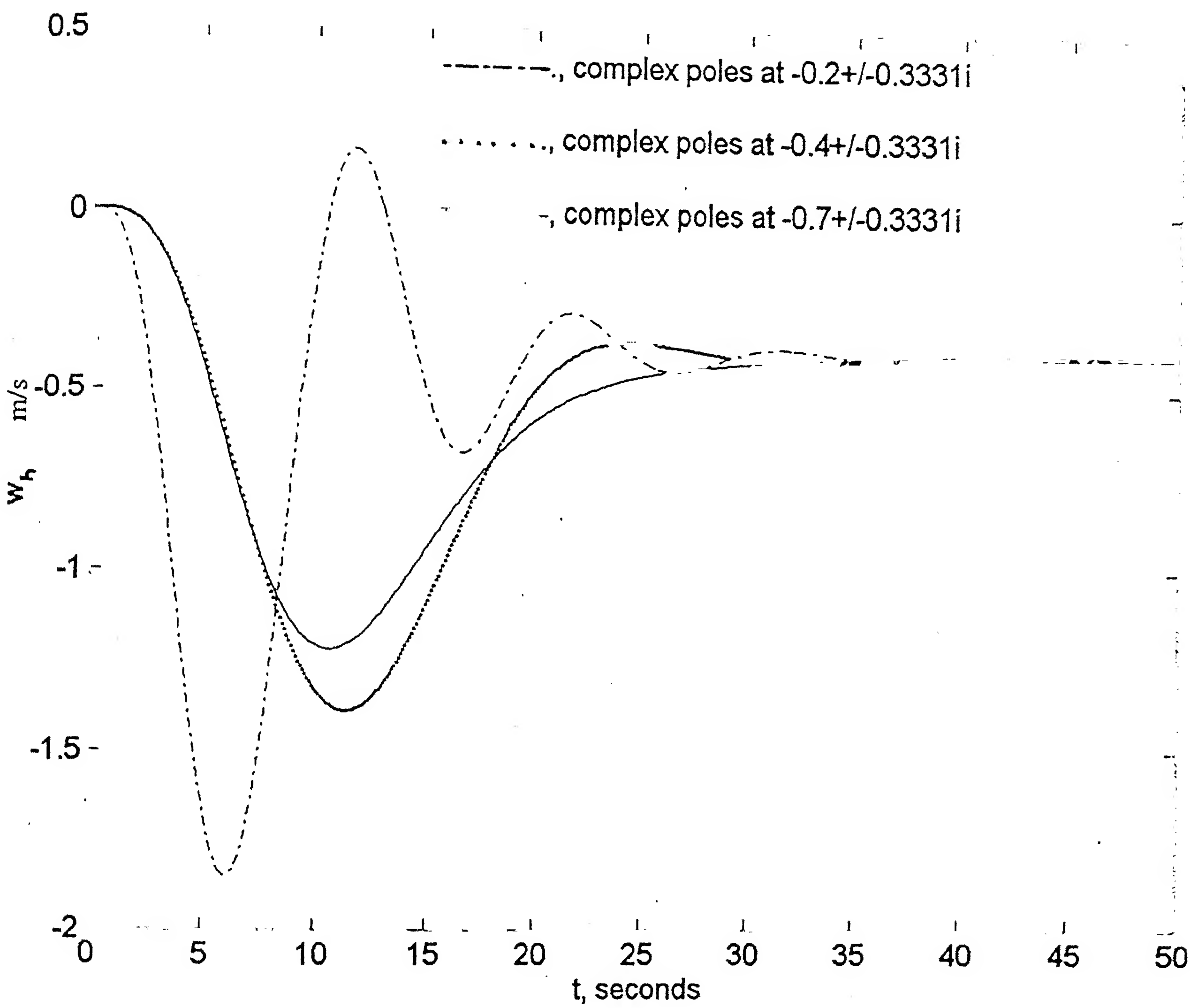

Figure 4.2: Response of w_h to a step i/p in θ_{1s}-Hover

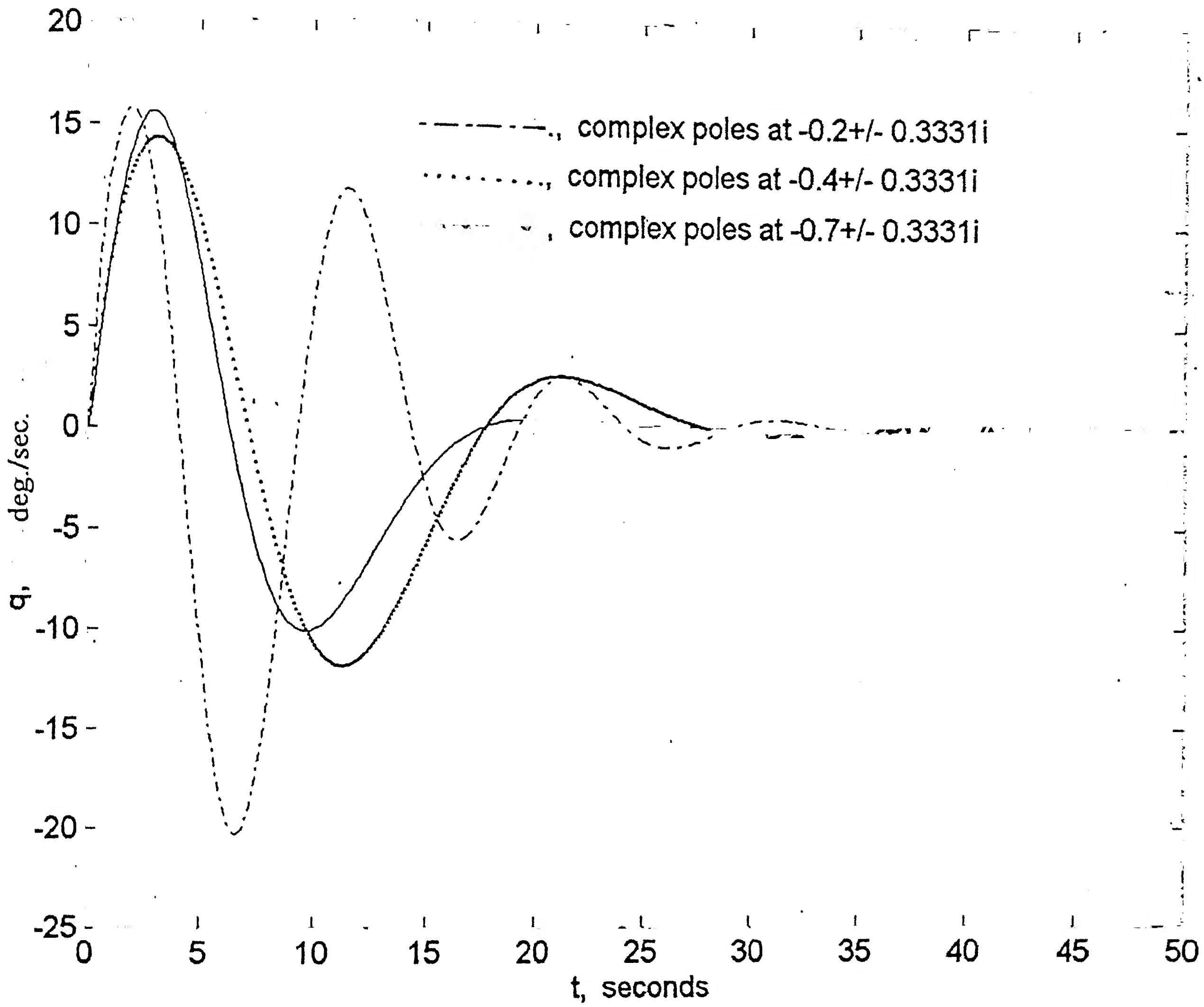

Figure 4.3: Response of q to a step i/p in θ_{1s}-Hover

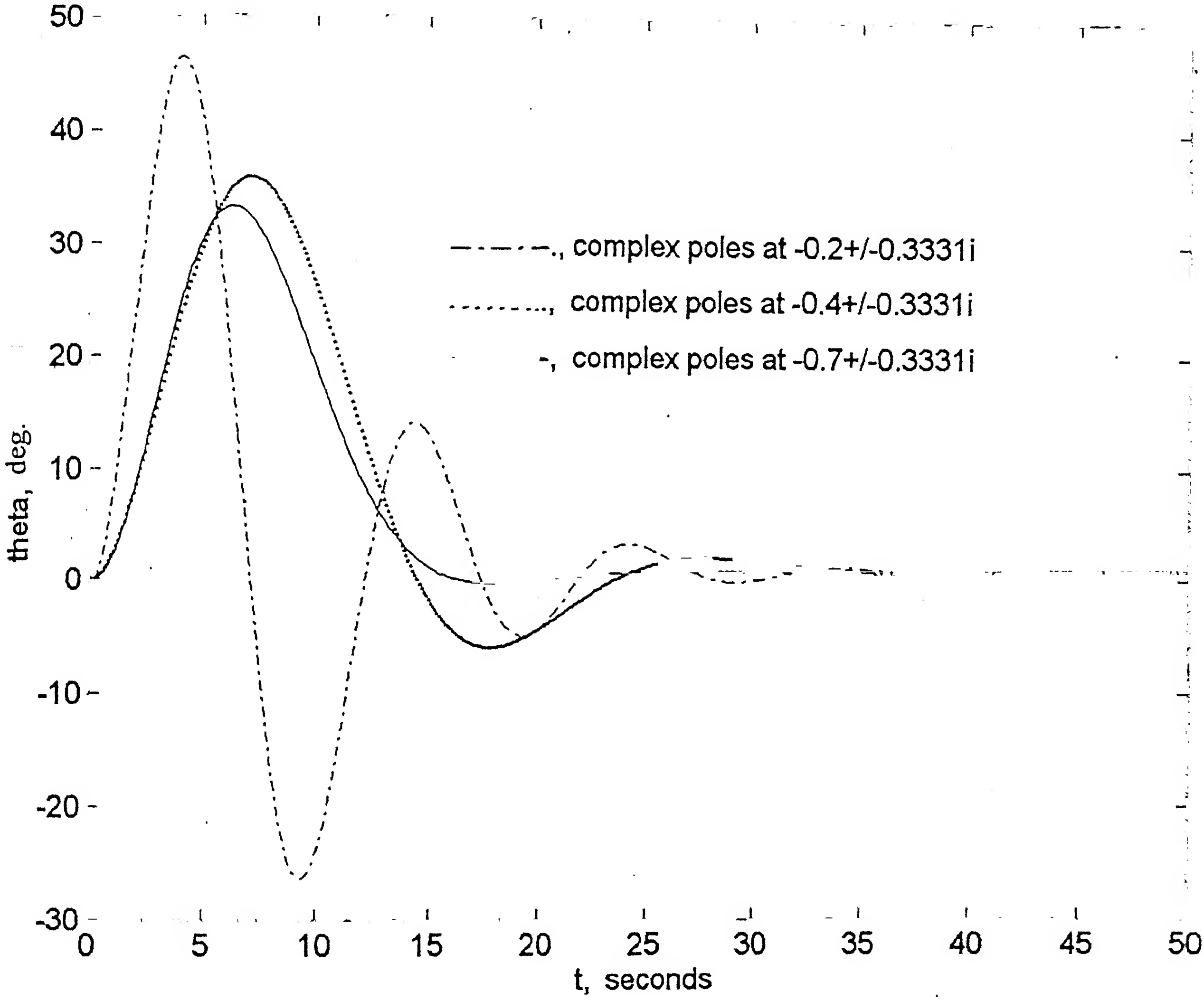

Figure 4.4: Response of θ to a step i/p in θ_{1s}-Hover

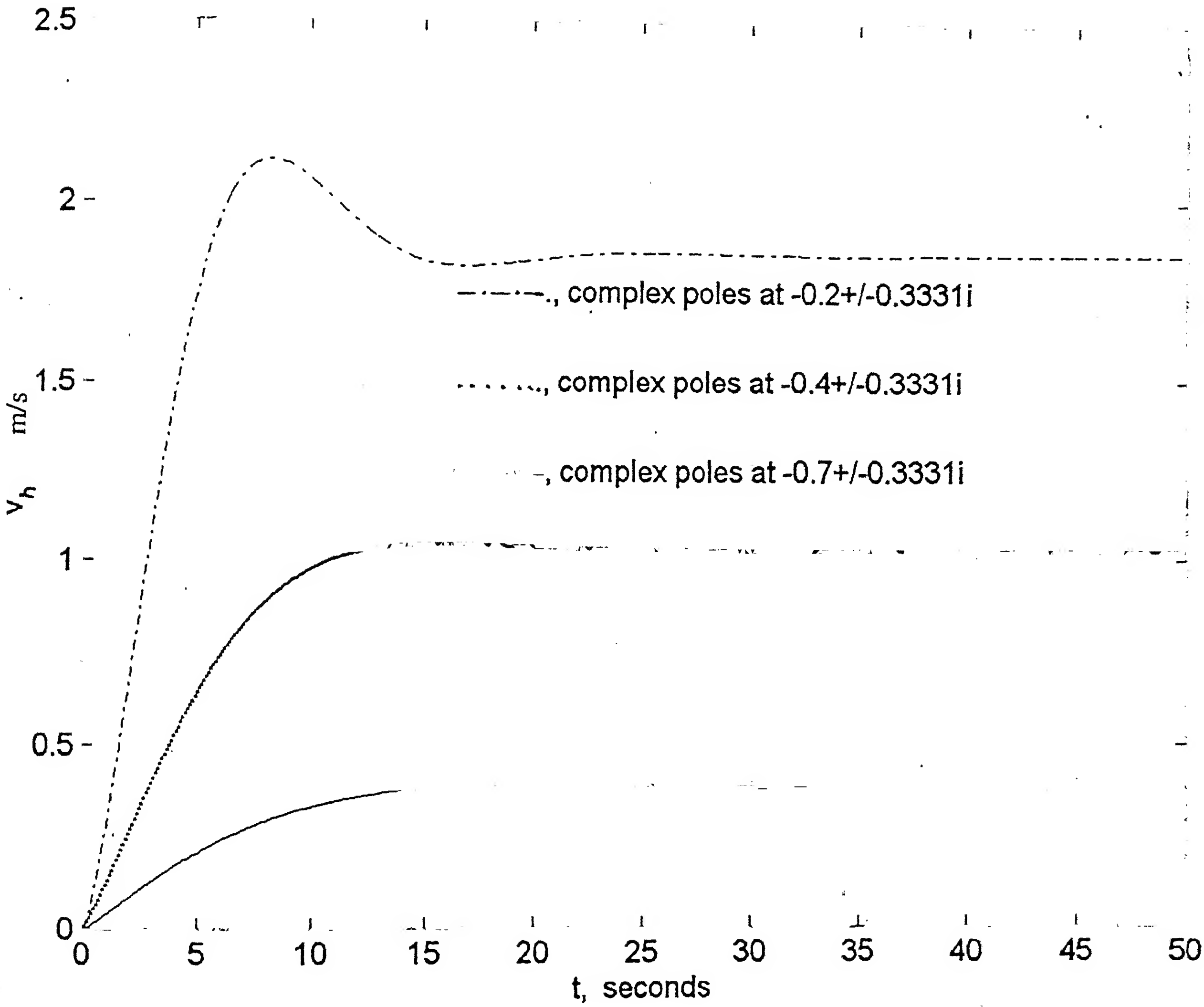

Figure 4.5: Response of v_h to a unit step in θ_{1c}-Hover

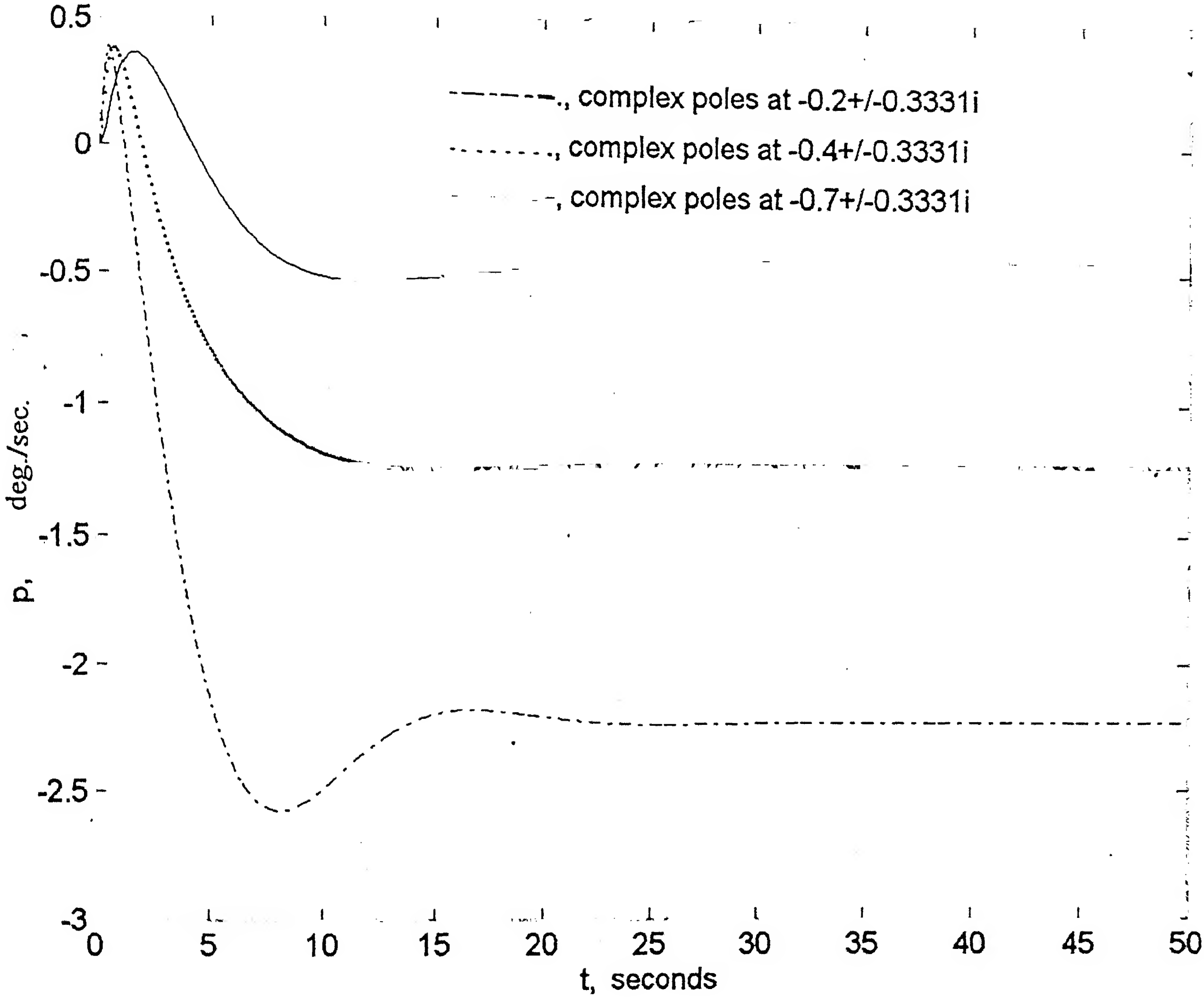

Figure 4.6: Response of p to a unit step in θ_{1c}-Hover

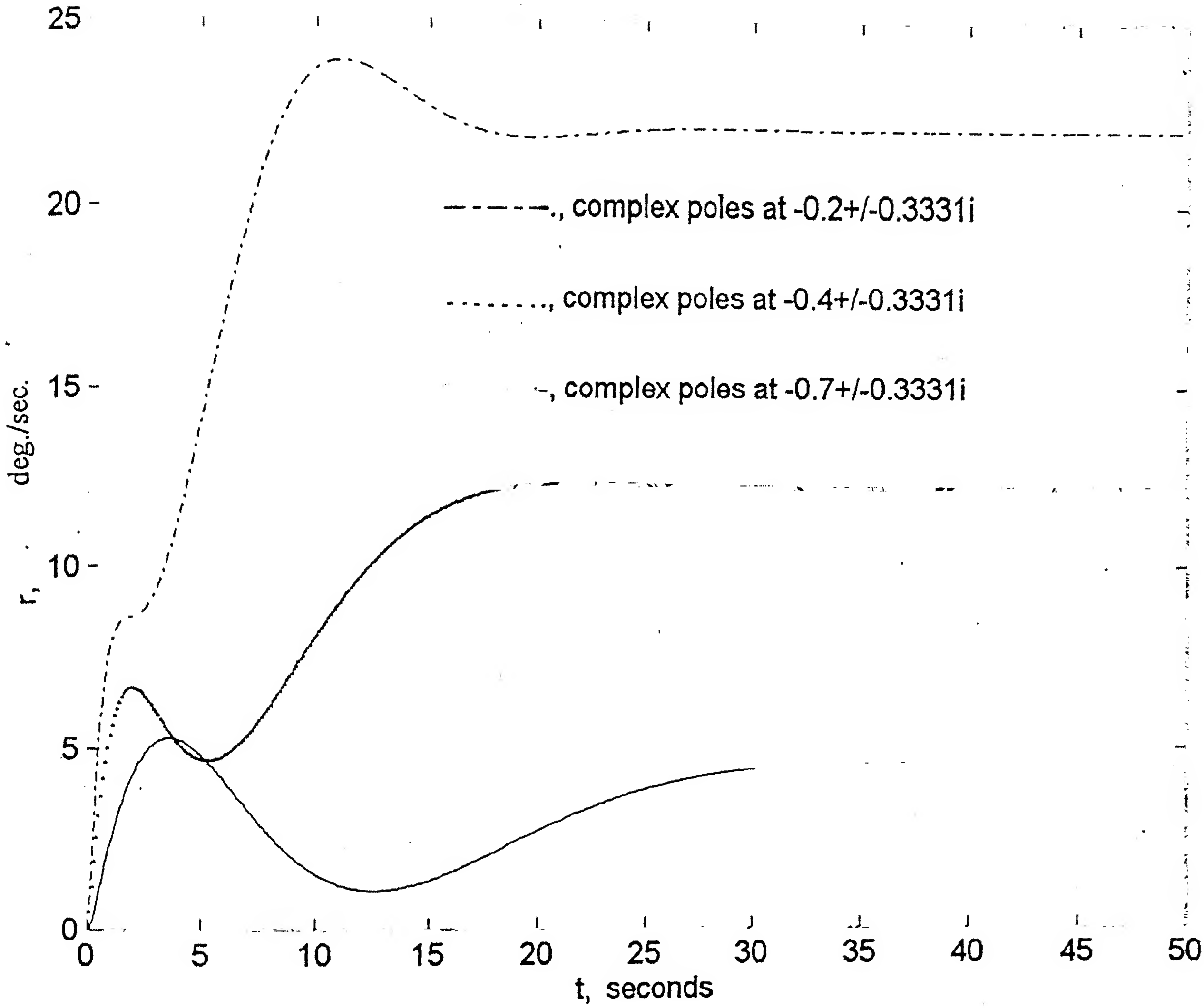

Figure 4.7: Response of r to a unit step in θ_{1c}-Hover

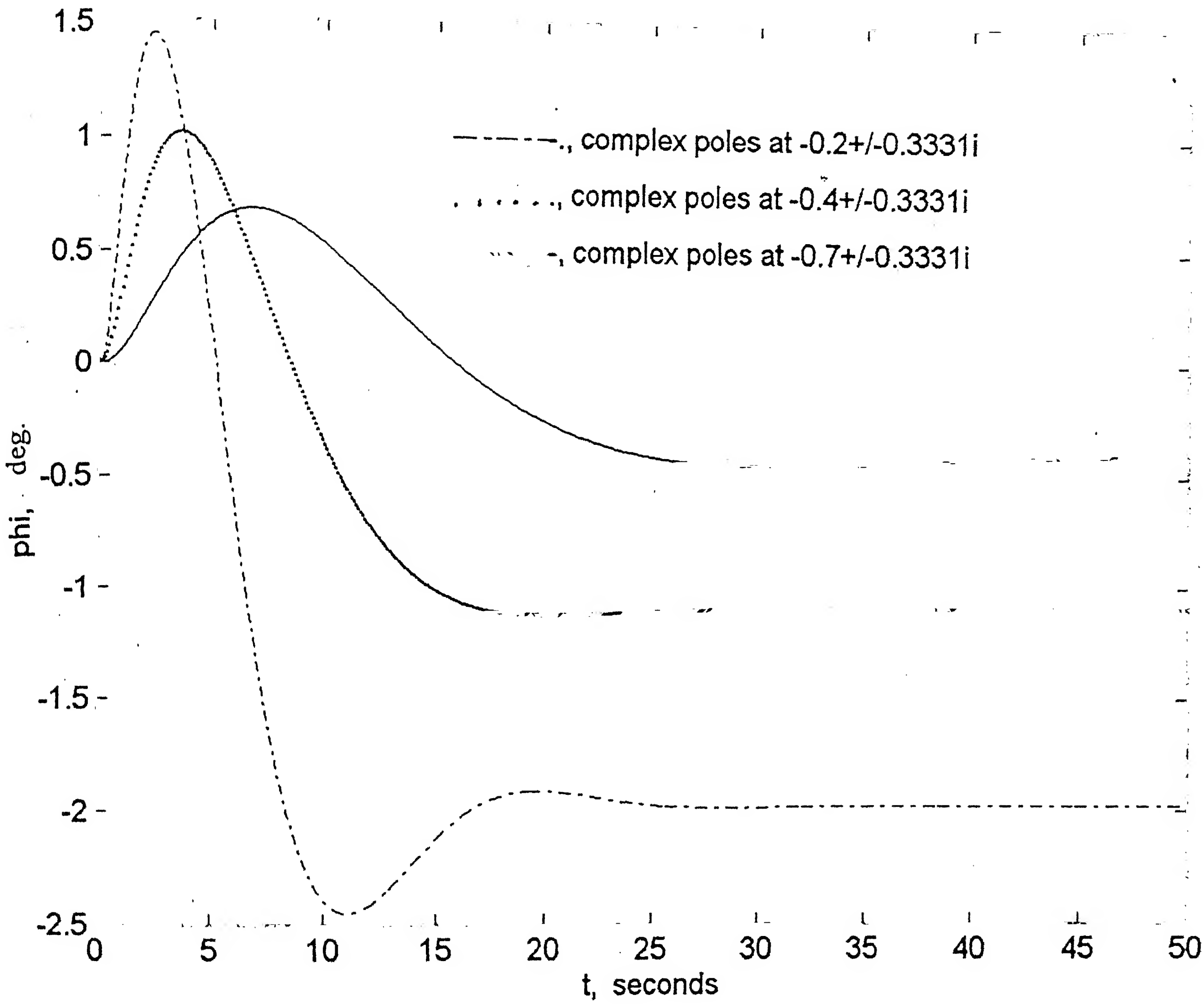

Figure 4.8: Response of ϕ to a unit step in θ_{1c}-Hover

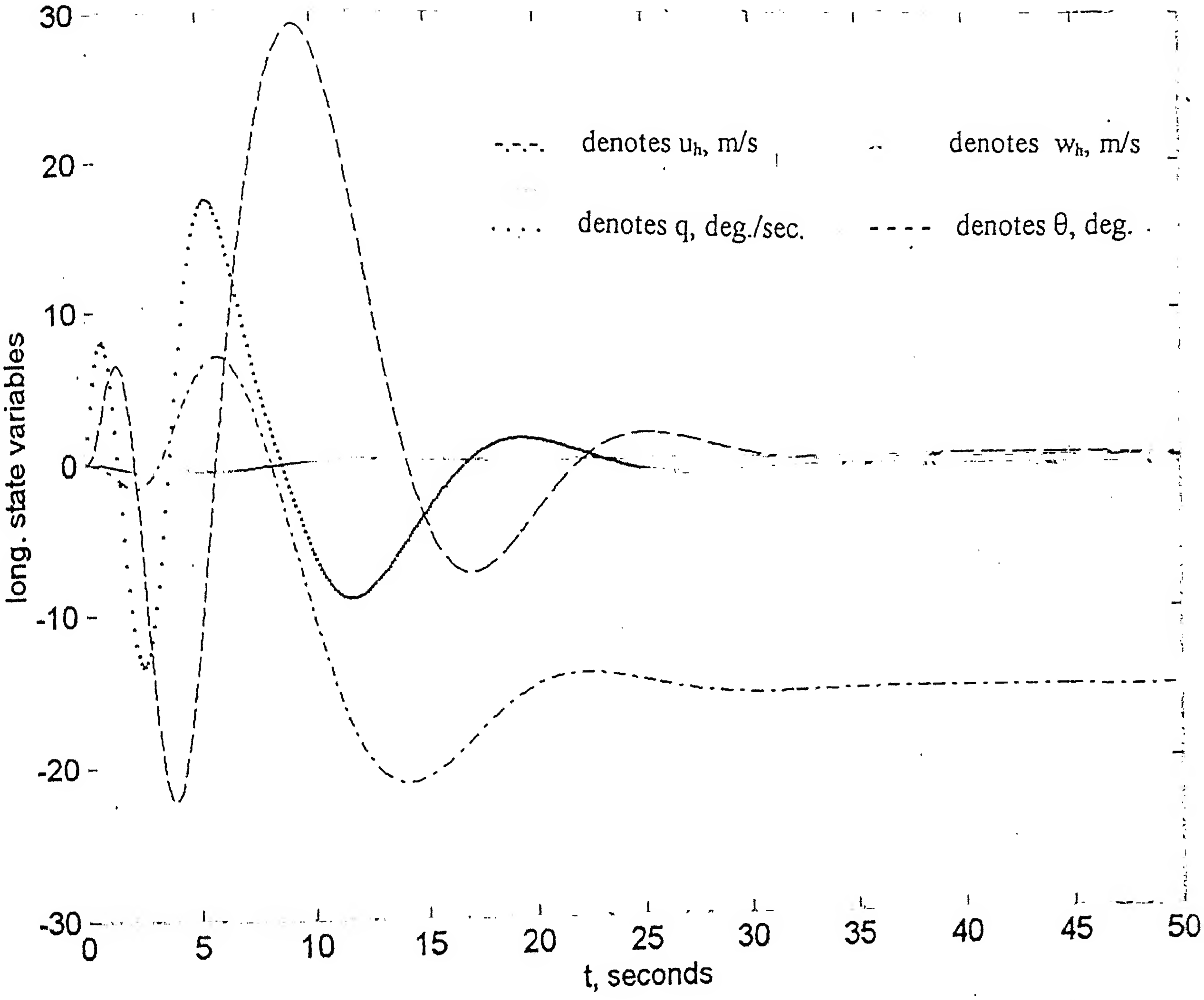

Figure 4.9: Response of all longitudinal state variables to a unit step in θ_{1s}-200kph

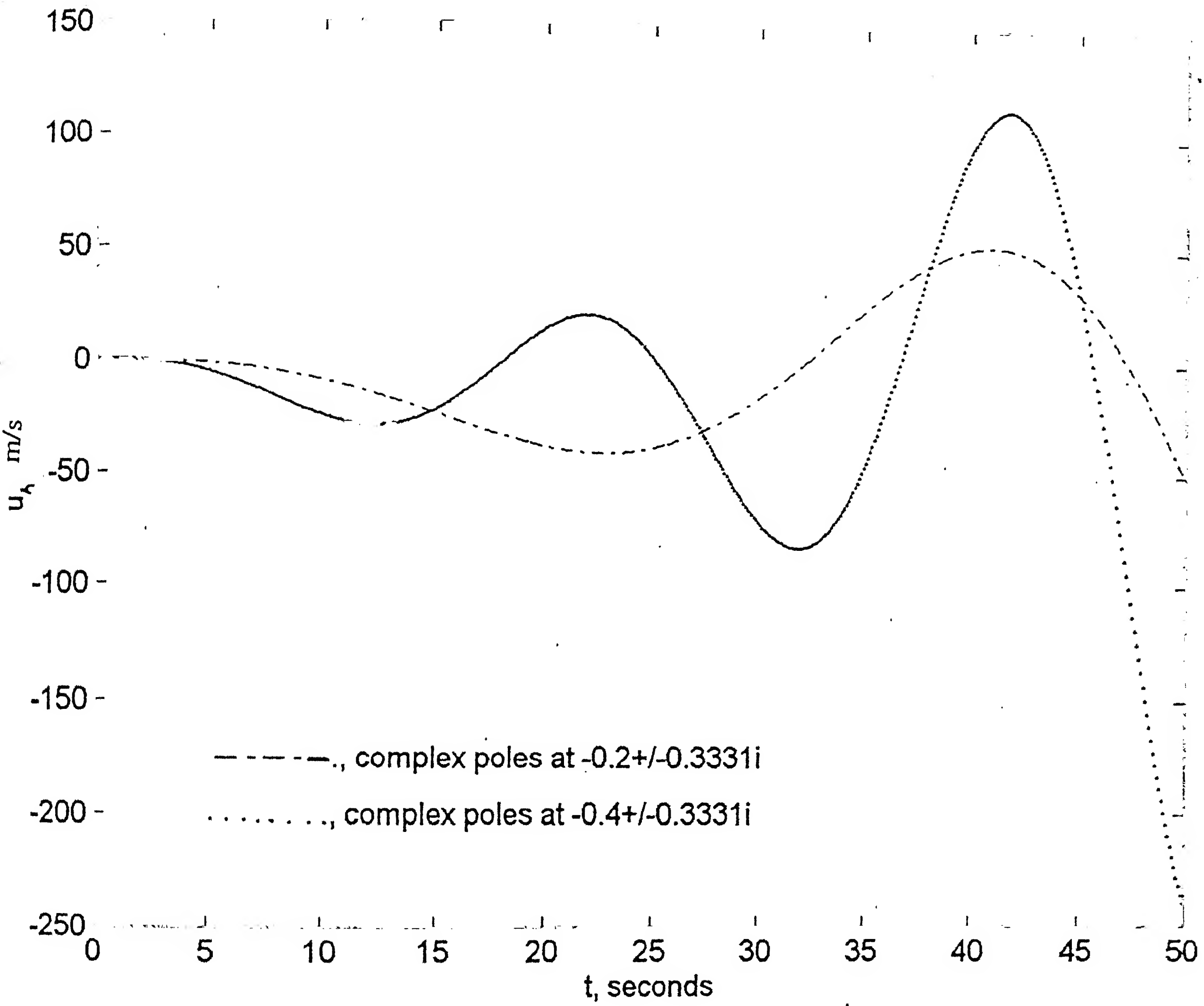

Figure 4.10: Response of u_h to unit step in both θ_{1s} and θ_{1c}-Hover

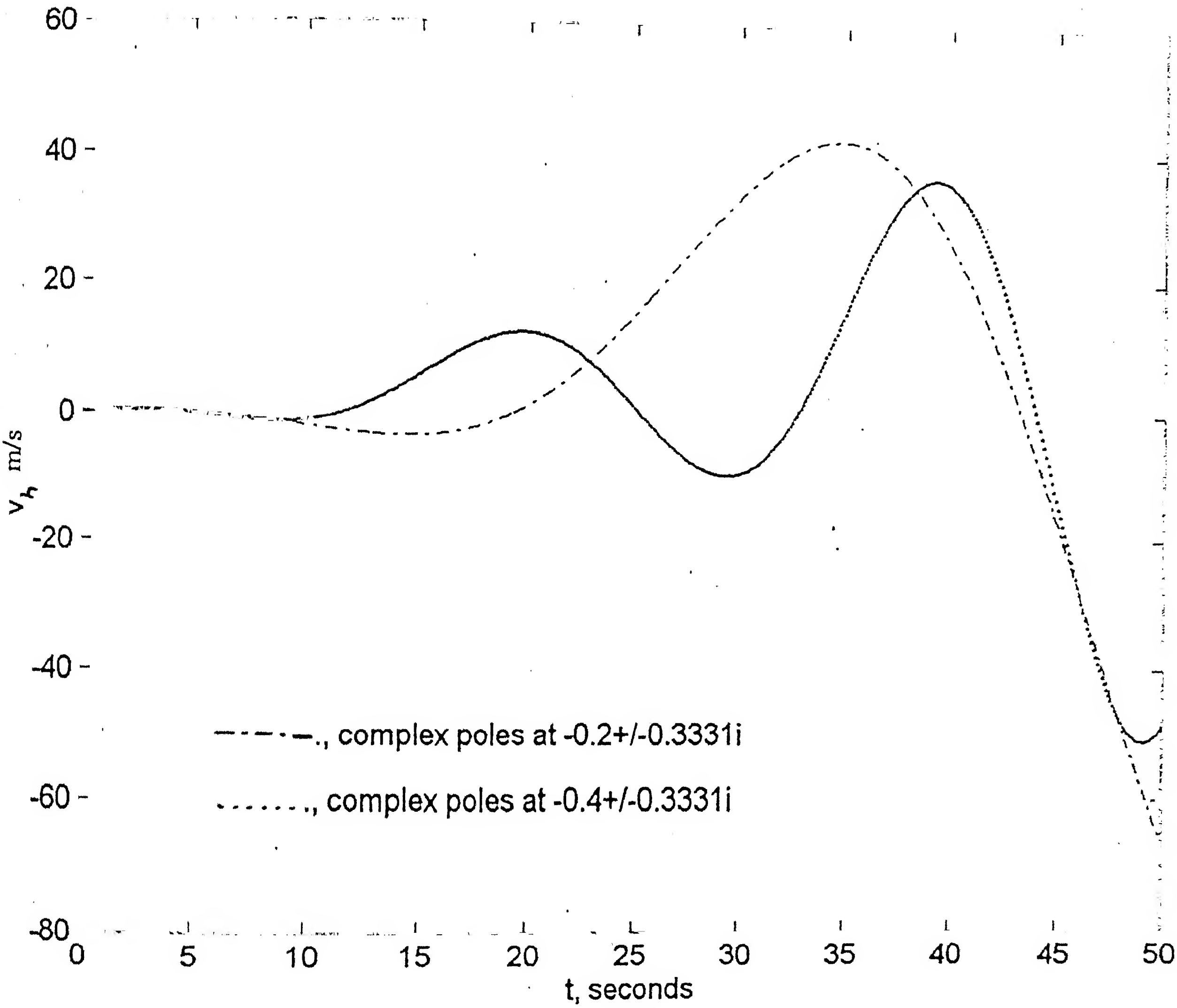

Figure 4.11: Response of v_h to unit step in both θ_{1s} and θ_{1c}-Hover

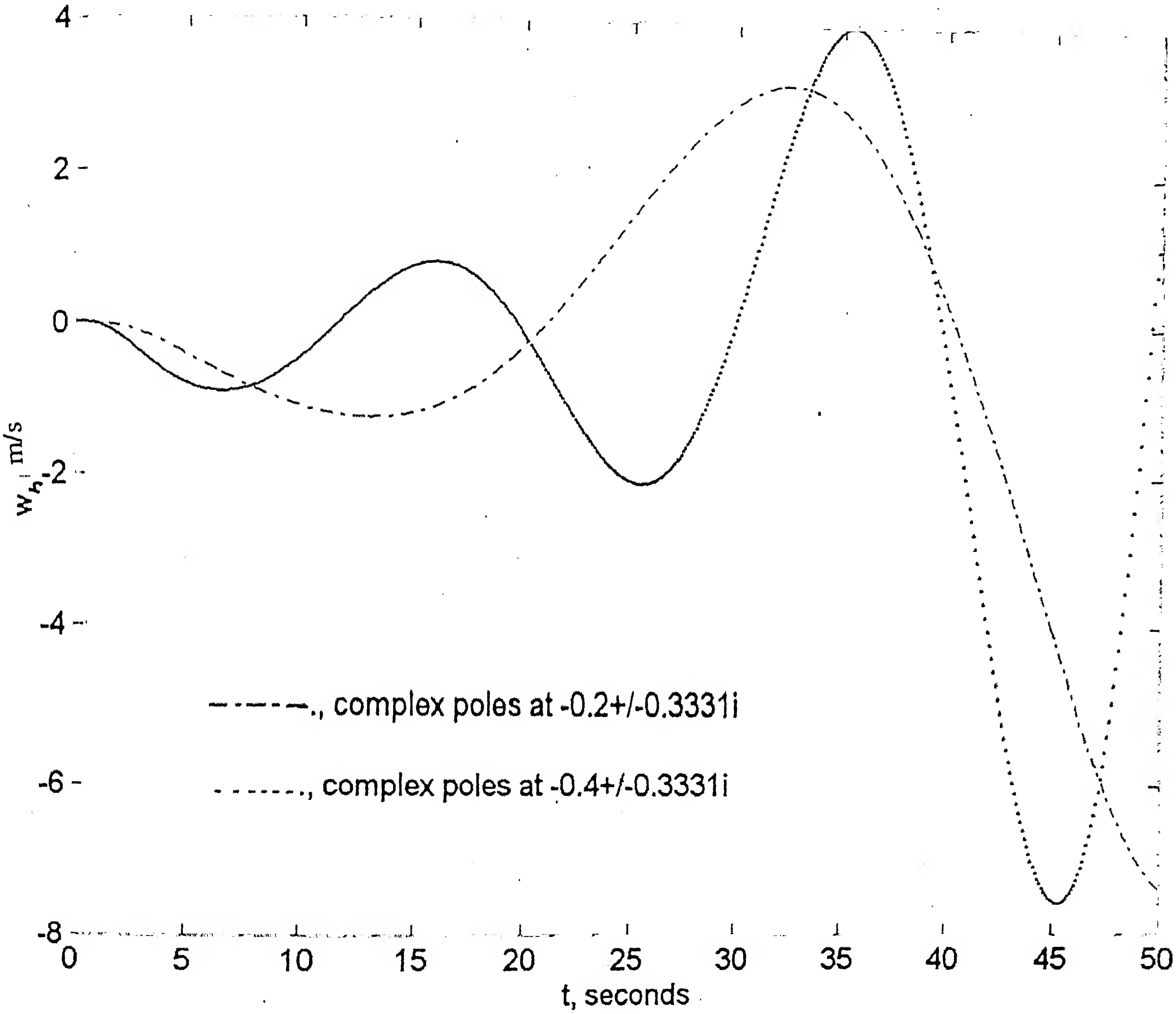

Figure 4.12: Response of w_h to unit step in both θ_{1s} and θ_{1c}-Hover

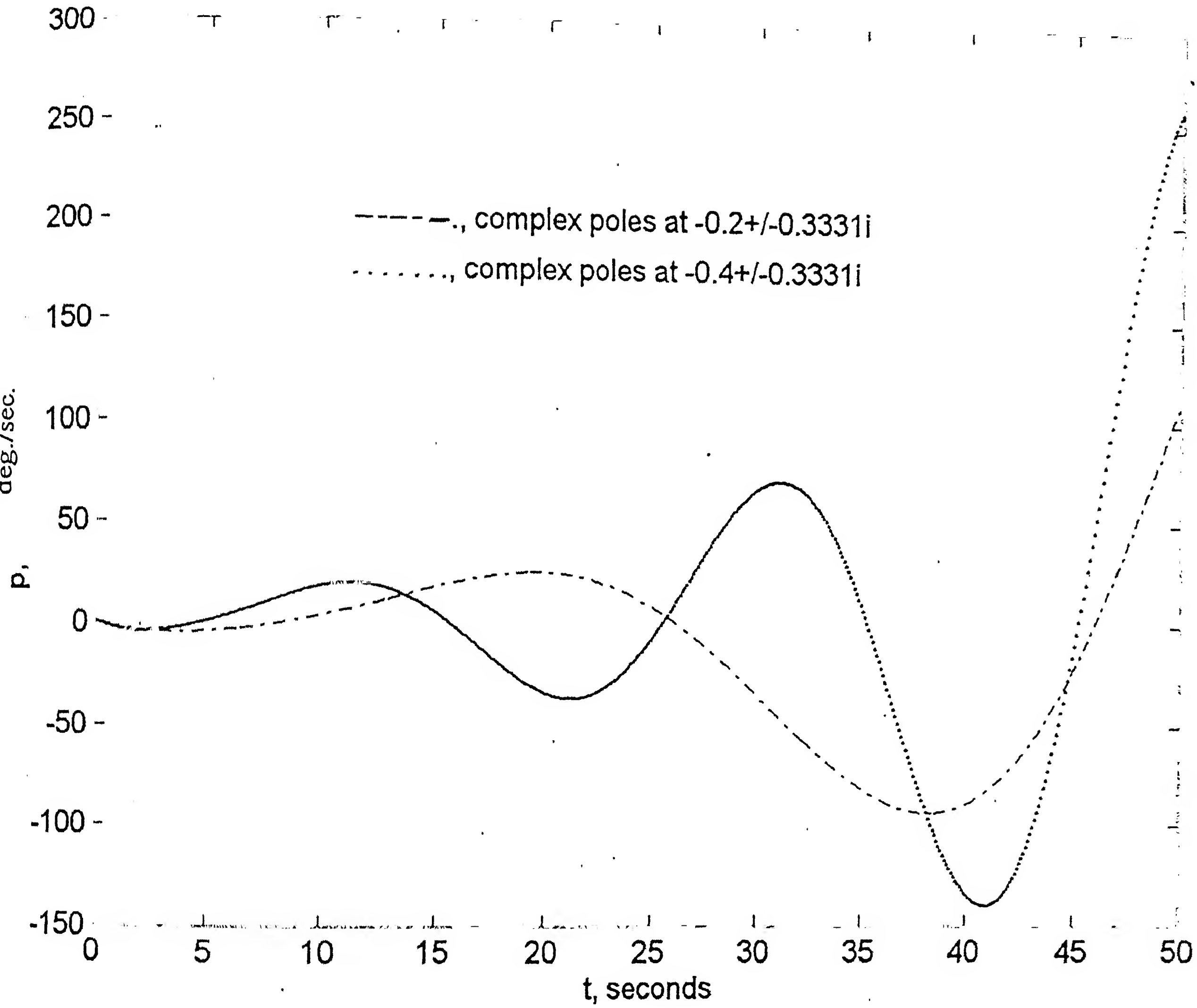

Figure 4.13: Response of p to unit step in both θ_{1s} and θ_{1c}-Hover

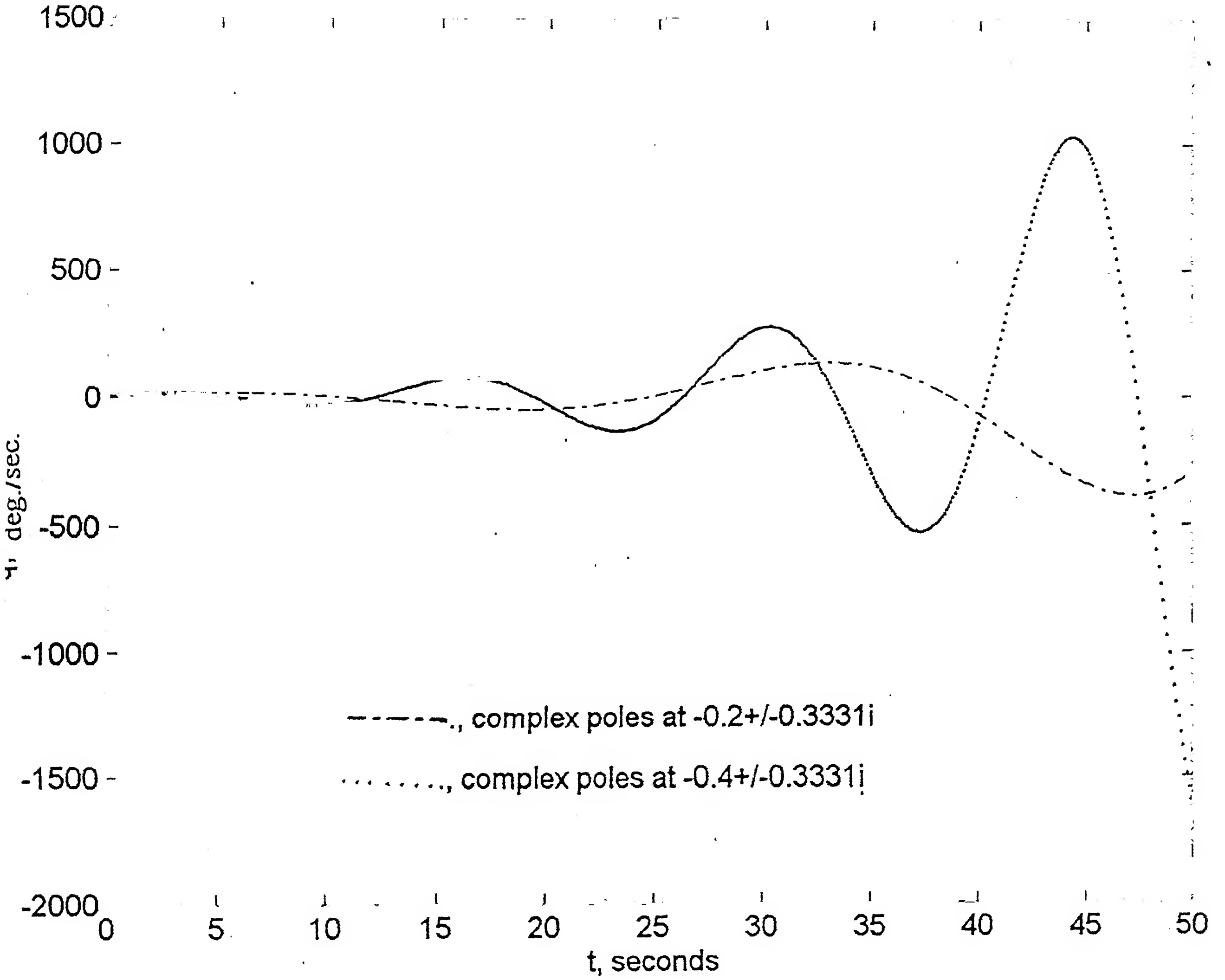

Figure 4.14: Response of q to unit step in both θ_{1s} and θ_{1c}-Hover

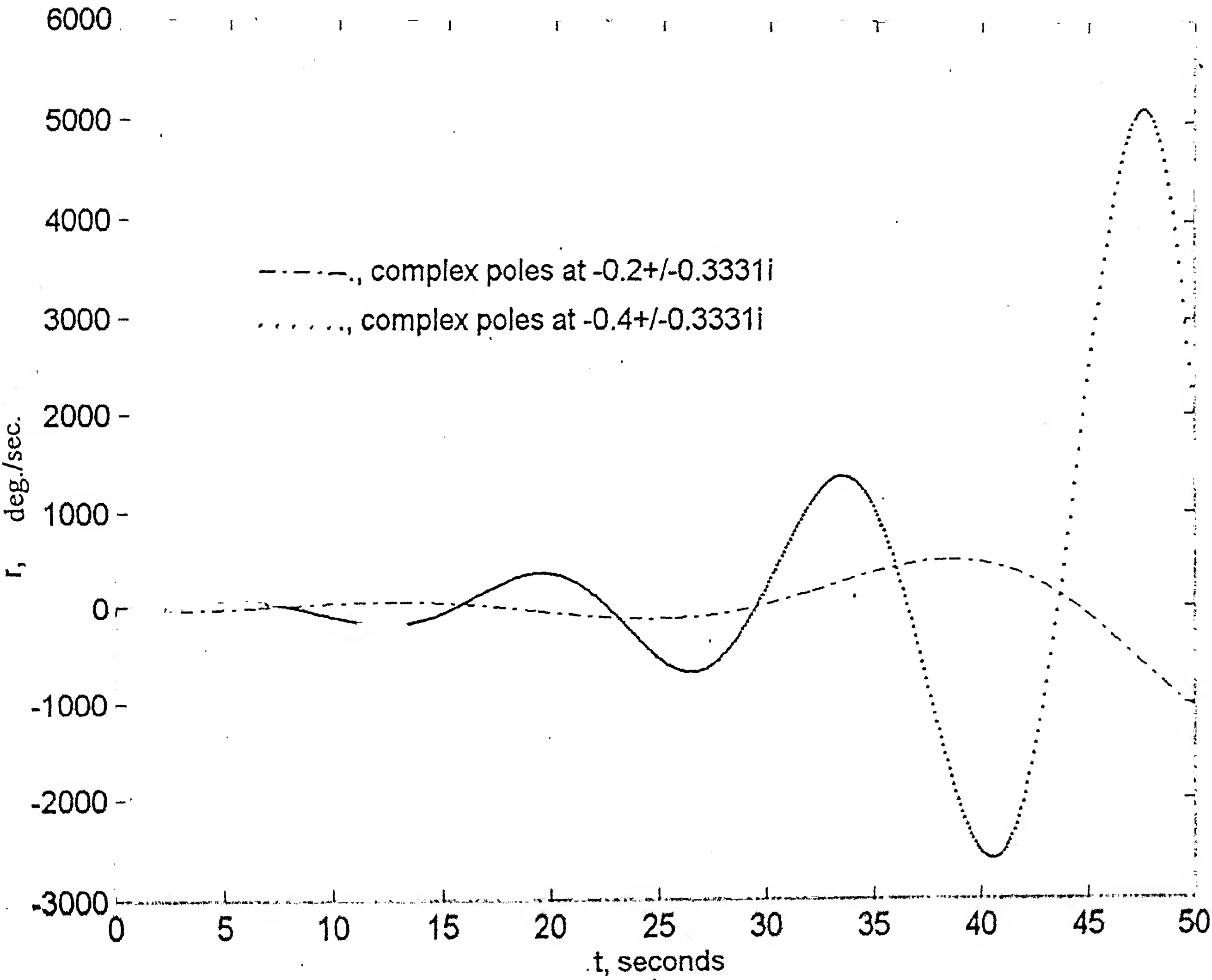

Figure 4.15: Response of r to unit step in both θ_{1s} and θ_{1c}-Hover

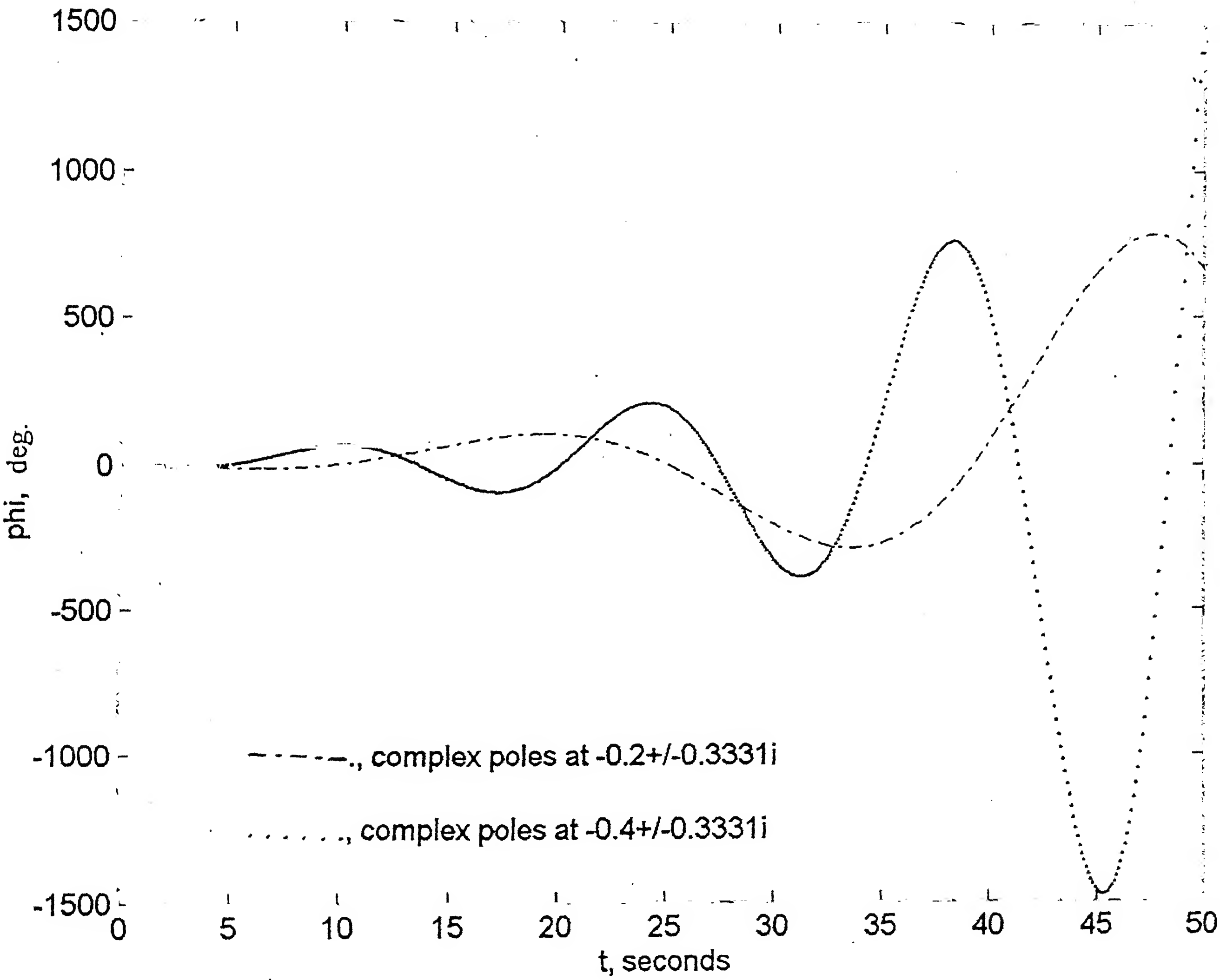

Figure 4.16: Response of ϕ to unit step in both θ_{1s} and θ_{1c}-Hover

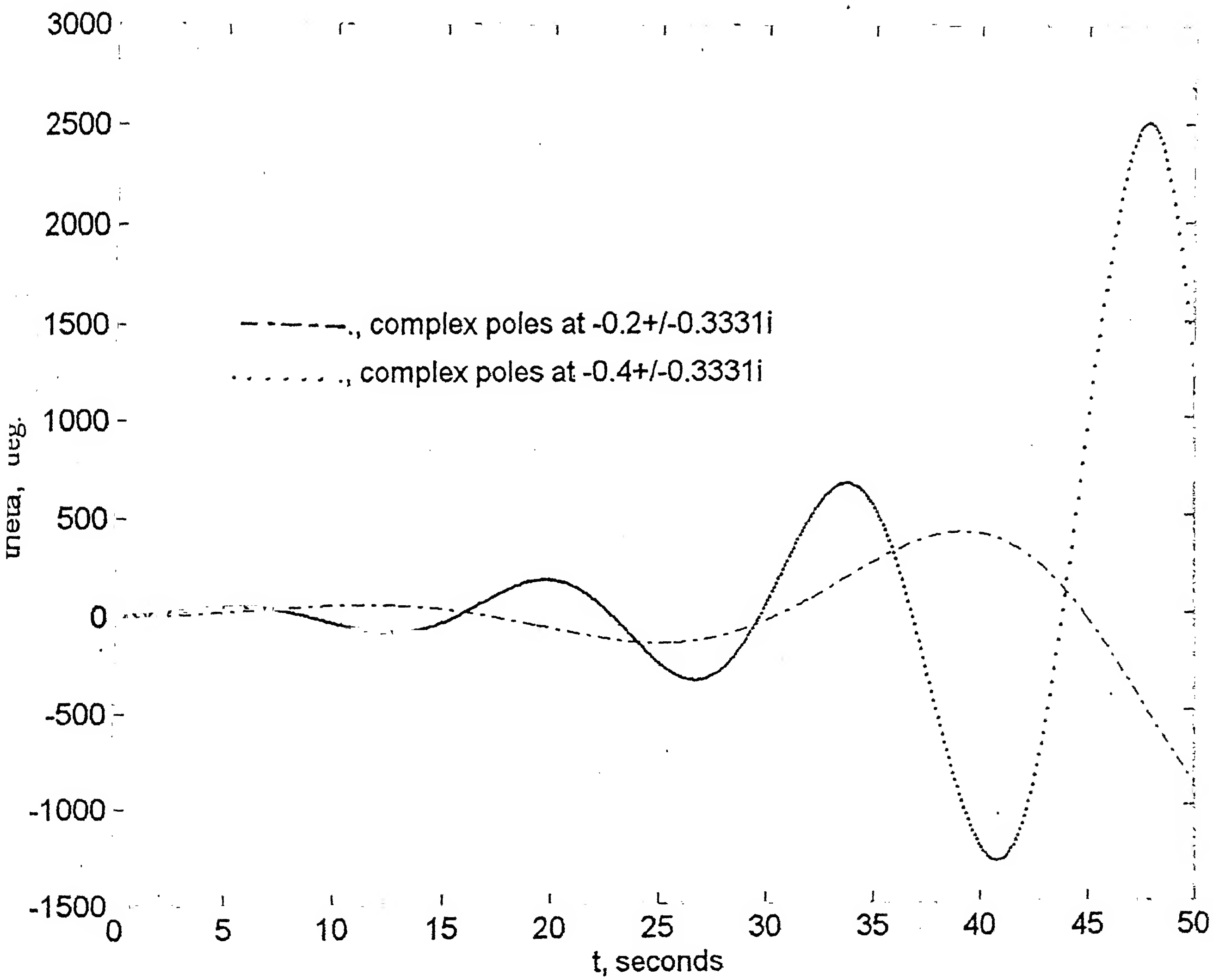

Figure 4.17: Response of θ to unit step in both θ_{1s} and θ_{1c}-Hover

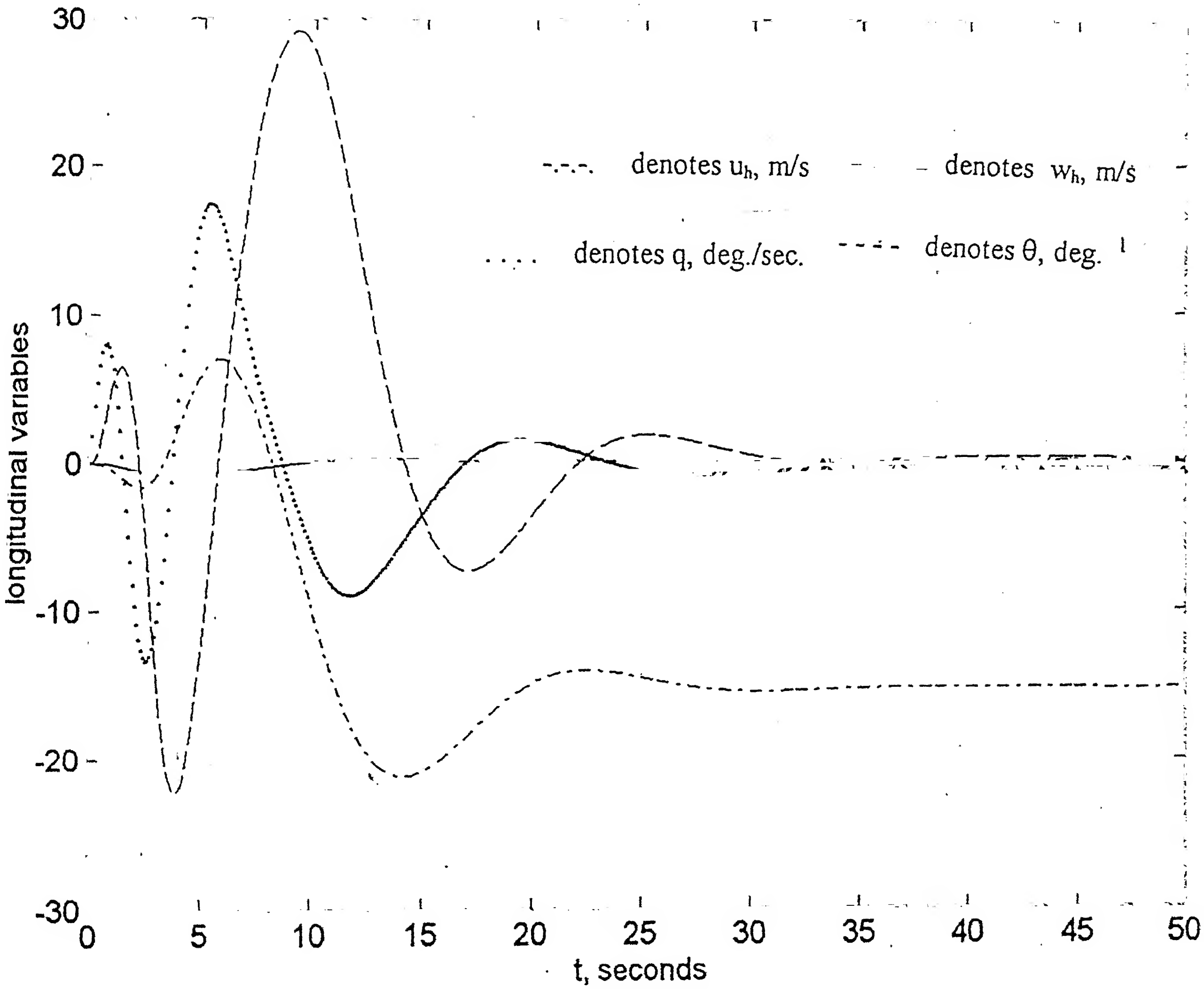

Figure 4.18: Response of the longitudinal state variables to unit step in both θ_{1s} and θ_{1c}-Hover

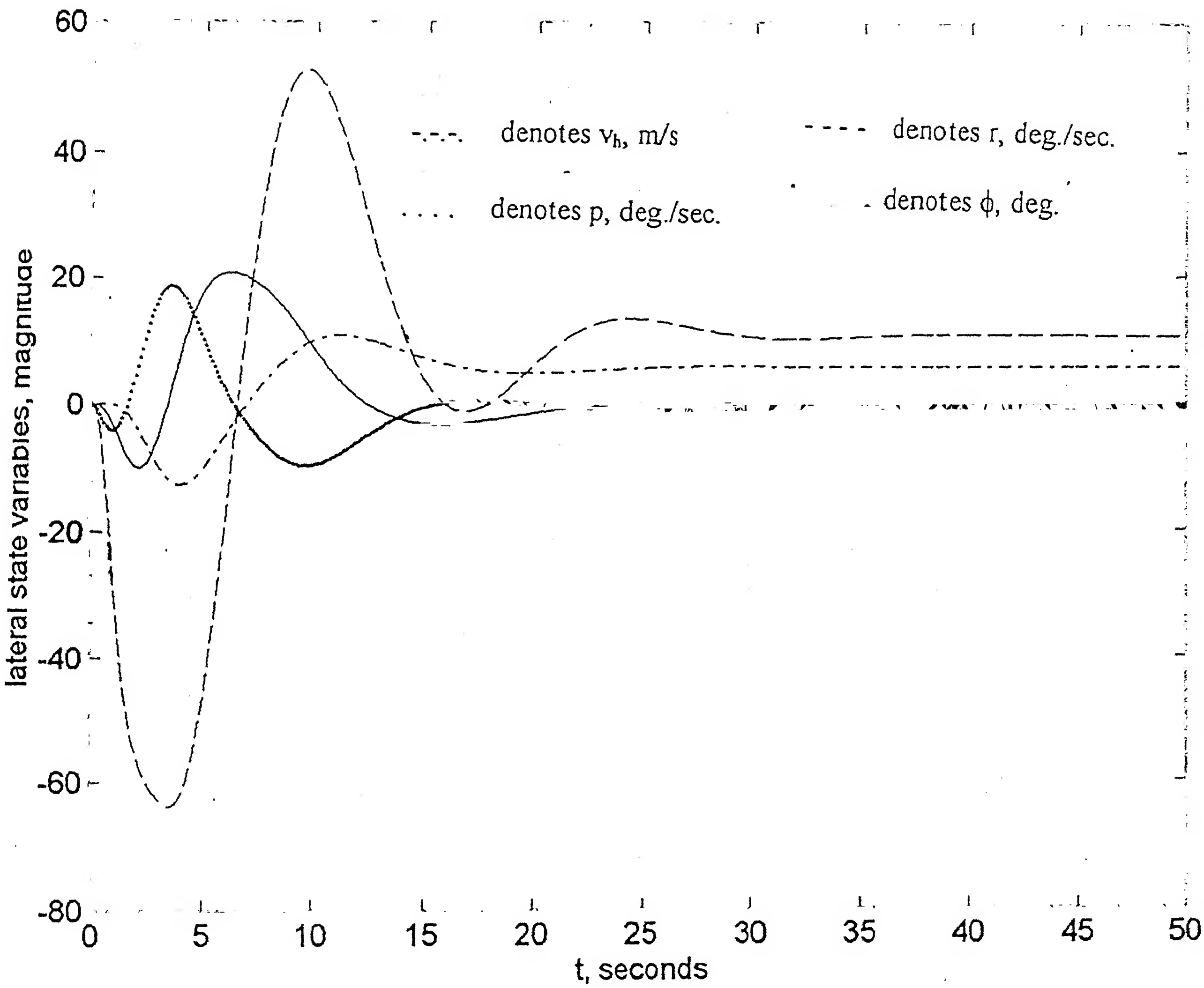

Figure 4.19: Response of the lateral state variables to unit step in both θ_{1s} and θ_{1c}-Hover

Chapter 5

Concluding Remarks

In this thesis, we discussed the application of H_∞ optimization techniques and factorization theory to the design of control laws for flight control system of a helicopter. In the first stage the characterization of all closed loop controllers which assigns the closed loop poles in the region for good handling qualities was carried out. This was developed using the standard controller parametrization of the factorization theory. All these controllers augment the stability of the vehicle as per MIL-H-8501A specification. In the next step, the forward compensator to augment the pilot input response is designed. Using the design methodology outlined in this chapter, two separate two degrees of freedom controllers are designed to give decoupled longitudinal and lateral control of the helicopter for two different flight conditions. The simulation study presented has shown that the control strategy proposed gives very good performance in the decoupled longitudinal and lateral axes. But the whole coupled system does not perform well with the decoupled longitudinal and lateral controllers. Hence using the same design strategy closed loop controller is designed for the whole coupled system.

Robust closed loop stabilization of the whole flight envelope, by including two extreme flight conditions ie,hover and full forward speed is investigated and is shown that for the uncertaity model chosen such a controller did not exist. Robust stabilizing controller is next designed at hover by considering the multiplicative description of plant uncertainty which is more realistic. The state space algorithm of [8] is used in designing the robust stabilizing controller.

5.1 Scope of future work

- One of the short comings of the designs so far undertaken has been that they have been restricted to one operating point. It is intended to develop a control system to cover wider operating envelope and this will entail undertaking designs at a number of operating points and investigating ways in which they may be scheduled or switched in a bumpless fashion.

- A very important area is that of actuator saturation, which can lead to a loss in the direction of control input signals and thus to a degradation in performance. In this thesis we have not considered the problem of actuator saturation. It would be very important to develop a controller conditioning strategy to ensure that performance is maintained in the face of actuator saturation.

- In this thesis, the controllers obtained are of high order(ie, 5 and above). The modern hardware does not pose serious limitations to implementing high order controllers. However it is still important to find reduced order controllers and confirming the performance of the closed loop system after reduction, due to their ease of implementation.

APPENDIX A

Parametrization Of All Stabilising Controllers

In this appendix the formula for obtaining a doubly coprime stable factorization of a plant transfer matrix P, is presented. Using this factorization, the formula for all stabilising compensators which guarantee poles of the closed loop system in a specified region of the complex plane, is also presented.

Consider the system transfer matrix P represented in a state space model as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

which is denoted in shorthand as

$$P = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = C(sI - A)^{-1}B + D$$

The doubly coprime factorization of P is defined as follows.

Definition:-

A doubly coprime factorization(DCF) of P over a closed region Ω in the complex plane is a factorization of P as

$$P = ND^{-1} = \tilde{D}^{-1}\tilde{N}$$

such that the following properties are satisfied.

1. N, D, $\tilde{N}$, $\tilde{D}$ have no poles in Ω.

2. There are transfer matrices X, Y, $\tilde{X}$, $\tilde{Y}$ which have no poles in Ω.

3. Y, $\tilde{Y}$ are square and nonsingular.

4. N, D, $\tilde{N}$, $\tilde{D}$, X, Y, $\tilde{X}$, $\tilde{Y}$ satisfy the identity

$$\begin{bmatrix} Y & X \\ -\tilde{N} & \tilde{D} \end{bmatrix} \begin{bmatrix} D & -\tilde{X} \\ N & \tilde{Y} \end{bmatrix} = I$$

The procedure for obtaining N, D, $\tilde{N}$, $\tilde{D}$ of a given plant P from its state space discription is given below. Suppose the given system

$$P = \left(\begin{array}{cc} A & B \\ C & D \end{array} \right)$$

is minimal. Select matrices K and H such that $A_K := A - BK$ and $A_H := A - HC$ have no eigen values in Ω. Then N, D, $\tilde{N}$, $\tilde{D}$, X, Y, $\tilde{X}$, $\tilde{Y}$ are given as [14]

$$\tilde{N} = \left(\begin{array}{cc} A_H & B - HD \\ C & D \end{array} \right)$$

$$\tilde{D} = \left(\begin{array}{cc} A_H & H \\ -C & I \end{array} \right)$$

$$N = \left(\begin{array}{cc} A_K & B \\ C - DK & D \end{array} \right)$$

$$D = \left(\begin{array}{cc} A_K & B \\ -K & I \end{array} \right)$$

$$X = \left(\begin{array}{cc} A_H & H \\ K & 0 \end{array} \right)$$

$$Y = \left(\begin{array}{cc} A_H & B - HD \\ K & I \end{array} \right)$$

$$\tilde{X} = \left(\begin{array}{cc} A_K & H \\ K & 0 \end{array} \right)$$

$$\tilde{Y} = \left(\begin{array}{cc} A_K & H \\ C - DK & I \end{array} \right)$$

Formula For All Stabilising Controllers

In the DCF above, if Ω is a compliment of an open connected domain Γ in the left half of the complex plane, then the factors N, D, $\tilde{N}$, $\tilde{D}$, X, Y, $\tilde{X}$, $\tilde{Y}$ are all stable transfer functions with all the poles in Γ. The well known formula which makes use of the DCF and expresses all the stabilising compensators C(s) such that the closed loop system of Fig.(5.1) with the transfer matrix from w to z has all its poles in Γ is given as follows. Let Q be any proper rational transfer matrix with no poles in Ω then

$$C = \{ (Y - Q\tilde{N})(X + Q\tilde{D})^{-1},\ det(Y - Q\tilde{N}) \not\equiv 0 \}$$

Figure 5.1: The Standard Block Diagram

$$= \{ (\tilde{X} + DQ)(\tilde{Y} - NQ)^{-1},\ det(\tilde{Y} - NQ) \not\equiv 0 \}$$

It is well known from the stable coprime factorization theory [14], that if C is a stabilising controller expressed as above then the closed loop transfer matrix of the system in Fig.(5.1) can be expressed in terms of the DCF and Q, where Q appears in the transfer matrix in an affine manner. This mathematical fact turned out to be of vital importance in the development of modern feedback design theory.

APPENDIX B

The Four Block Control Problem

The standard 4-block problem is shown in the figure below. In the modern four block approach to control system design the plant G, to be controlled is assumed to have the structure as shown in Fig.(5.2).

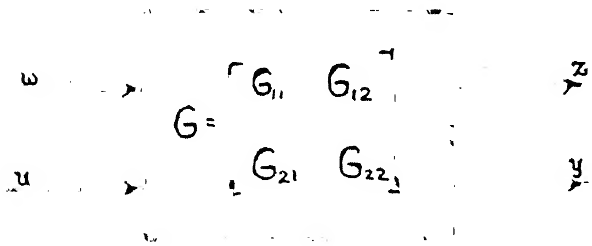

Figure 5.2: The Generalized Plant

where w is the exogeneous input, typically consisting of command signals, disturbances and sensor noises. u is the control signal. z is the output to be controlled, its components typically being tracking errors, filtered actuator signals etc. y is the measured output. A feedback controller C is attached to the plant to form the feedback system shown in Fig.(5.3).

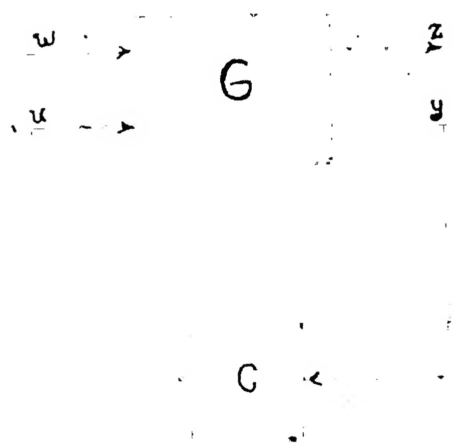

Figure 5.3: The Standard Problem

For G given by the four block transfer matrix

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

The equations of the feedback system are then given by

$$\begin{aligned} z &= G_{11}w + G_{12}u \\ y &= G_{21}w + G_{22}u \\ u &= -Cy \end{aligned}$$

Hence for a well posed system(ie, when det(I+GC)$\not\equiv$ 0), the transfer matrix denoting the response z due to exogenous input w is obtained as

$$y = T_{zw}w$$

where

$$T_{zw} = G_{11} - G_{12}C(I + G_{22}C)^{-1}G_{21}$$

Standard problem:-

The standard problem is to find a real rational proper C to minimize the H_∞ norm of T_{zw}, the transfer matrix from w to z under the consraint that C stabilizes G. All the requirements like robustness, disturbance rejection, tracking accuracy etc can be captured in the transfer matrix between w and z. The standard problem above is quite well developed and a state space algorithmic procedure is available for its solution. One such procedure is detailed below.

Glover-Doyle state space algorithm

The 4-block plant G is considered in the state space form

$$G(s) = \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix}$$

with the following assumptions.

1. (A, B_2) is stabilizable, (C_2, A) is detectable.

2. D_{12} is full column rank, D_{21} is full row rank.

3. $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full coloumn rank $\forall\omega$.

4. $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank $\forall\omega$.

Define the two Hamiltonian matrices as

$$H_\infty(\gamma) = \begin{bmatrix} A & 0 \\ -C_1^TC_1 & -A^T \end{bmatrix} - \begin{bmatrix} B \\ -C_1^TD_{1.} \end{bmatrix} BR^{-1} \begin{bmatrix} D_{1.}^TC_1 & B^T \end{bmatrix}$$

$$J_\infty(\gamma) = \begin{bmatrix} A^T & 0 \\ -B_1B_1^T & -A \end{bmatrix} - \begin{bmatrix} C^T \\ -B_1D_{.1}^T \end{bmatrix} \tilde{R}^{-1} \begin{bmatrix} D_{.1}B_1^T & C \end{bmatrix}$$

where

$$D_{1.} = \begin{bmatrix} D_{11} & D_{12} \end{bmatrix}$$

$$D_{.1} = \begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix}$$

and

$$R = D_{1.}^TD_{1.} - \begin{bmatrix} \gamma^2 I_{m1} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{R} = D_{.1}D_{.1}^T - \begin{bmatrix} \gamma^2 I_{p1} & 0 \\ 0 & 0 \end{bmatrix}$$

D_{11} is partitioned as

$$D_{11} = \begin{bmatrix} D_{1111} & D_{1112} \\ D_{1121} & D_{1122} \end{bmatrix}$$

Let A, Q, R be real nxn matrices with Q and R symmetric. Define the 2nx2n Hamiltonian matrix

$$H := \begin{bmatrix} A & R \\ Q & -A^T \end{bmatrix}$$

The dom(Ric) consists of Hamiltonian matrices with two properties, stability property and complementarity property [13]. The conditions for H to belong to dom(Ric) are as

follows. H should have no imaginary eigen values, R should be either positive semidefinite or negative semidefinite and (A,R) stabilizable.

Theorm:-

There exists stabilizing controllers such that $\| T_{zw} \| < \gamma$ if and only if the following conditions hold:

1. $\gamma > max\left(\bar{\sigma}[D_{1111}\ D_{1112}],\ \bar{\sigma}[D_{1111}^T\ D_{1121}^T]\right)$

2. $H_\infty(\gamma) \in$ dom(Ric) and $X(\gamma) :=$Ric$[H_\infty] \geq 0$
   $J_\infty(\gamma) \in$ dom(Ric) and $Y(\gamma) :=$Ric$[J_\infty] \geq 0$

3. $\rho[X(\gamma)Y(\gamma)] < \gamma^2$

When these conditions hold, one such controller is

$$K(s) = \begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix}$$

where

$$\begin{aligned}
\hat{D} &= -D_{1121}\ D_{1111}^T(\gamma^2 I - D_{1111}D_{1111}^T)^{-1}D_{1112} - D_{1122} \\
\hat{C} &= [F_2 - \hat{D}(C_2 + F_{12})]Z \\
\hat{B} &= -H_2 + (B_2 + H_{12})\hat{D} \\
\hat{A} &= A + HC + (B_2 + H_{12})\hat{C} \\
Z &:= (I - \gamma^{-2}YX)^{-1} \\
F^T &= [F_{11}^T\ F_{12}^T\ F_2^T] = -(XB + C_1^T D_{1.})R^{-1} \\
H &= [H_{11}\ H_{12}\ H_2] = -(YC^T + B_1 D_{.1}^T)\tilde{R}^{-1}
\end{aligned}$$

APPENDIX C

Robust Stabilization

The robust stability condition, in terms of the standard 4-block problem require $\| T_{zw} \|_\infty < 1$. The solvability of this standard problem using model matching theory i given below.

The transfer matrix between w and z can be written as [6]

$$T_{zw} = T_1 - T_2 Q T_3,$$

with C(s) given in Appendix A. T_1, T_2 and T_3 are defined as follows. Let the standar set up of Fig.(5.1) in Appendix A has state space realization

$$G(s) = \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix}$$

Choose F and H such that $A_F := A - BF$ and $A_H := A - HC$ are stable. Then T_1, and T_3 are given by

$$T_1(s) = \begin{pmatrix} \underline{A} & \underline{B} \\ \underline{C} & D_{11} \end{pmatrix}$$

$$\underline{A} = \begin{bmatrix} A_F & B_2 F \\ 0 & A_H \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} B_1 \\ B_1 - H D_{21} \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} C_1 - D_{12} F & D_{12} F \end{bmatrix}$$

$$T_2(s) = \begin{pmatrix} A_F & B_2 \\ C_1 - D_{12} F & D_{12} \end{pmatrix}$$

$$T_3(s) = \begin{pmatrix} A_H & B_1 - H D_{21} \\ C_2 & D_{21} \end{pmatrix}$$

Let z_i, $i = 1, 2, 3, \ldots, k$ denotes the right half plane transmission zeroes of T_2 and T_3. Then inorder for the robust stabilization problem to be solvable, T_1 must satisfy the condition, $\| T_1(z_i) \| < 1 \; \forall i = 1, 2, 3, \ldots, k$.

APPENDIX D

Model of The Helicopter at Different Flight Conditions

The model of the helicopter at hover, 200kph and 330kph are given below.

Model at Hover

System Matrix (8X8)

$$A = \begin{bmatrix} -0.1567E-01 & 0.1900E-02 & 0.3119E-01 & -0.7445E-02 & 0.5274E-02 & -0.1587E-02 & -0.4785E-11 & -0.1703E+00 \\ -0.5833E-02 & -0.0007E-01 & 0.1331E+01 & -0.8362E-02 & 0.6488E-01 & 0.9020E-02 & 0.1701E+00 & 0.8476E-03 \\ 0.4771E-02 & -0.5832E-02 & -0.3346E+00 & -0.1876E-03 & 0.4477E-02 & 0.1079E-01 & 0.8369E-02 & -0.1723E-01 \\ -0.2627E+01 & -0.6690E+01 & -0.3014E+02 & -0.5162E+01 & -0.2531E+01 & -0.4261E-01 & -0.3097E-07 & -0.2115E-00 \\ 0.9390E+00 & -0.4781E+00 & -0.9053E-01 & 0.1959E+00 & -0.1322E+01 & 0.5320E-01 & -0.9409E-09 & -0.1140E-07 \\ -0.2003E+00 & 0.5671E+01 & -0.1170E+03 & -0.5650E+00 & -0.6537E+01 & -0.9319E+00 & -0.1935E-07 & -0.5311E-[illegible] \\ 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.1000E+01 & -0.4977E-02 & 0.1012E+00 & 0.0000E+00 & 0.0000E+00 \\ 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.9900E+00 & 0.4915E-01 & 0.0000E+00 & 0.0000E+00 \end{bmatrix}$$

Control Matrix

$$B = \begin{bmatrix} 0.2264264E-01 & -0.2060993E+00 \\ 0.1950103E+00 & 0.1500457E-01 \\ -0.3645610E-01 & -0.1030504E-01 \\ 0.0306200E+02 & -0.2353194E+01 \\ 0.1333310E+01 & 0.2154925E+02 \\ 0.1430312E+02 & -0.3720501E+00 \\ 0.0000000E+00 & 0.0000000E+00 \\ 0.0000000E+00 & 0.0000000E+00 \end{bmatrix}$$

Model at 200kph

System Matrix (8X8)

$$A = \begin{bmatrix} -0.4132E-01 & 0.5451-03 & 0.2400E-01 & -0.0120E-02 & 0.3598E-01 & -0.9600E-04 & -0.2251E-11 & -0.1711E+00 \\ -0.7167E-03 & -0.2313E+00 & -0.2185E-01 & -0.4072E-01 & -0.5673.E-02 & -0.9036E+00 & 0.1716E+00 & -0.1355E-03 \\ 0.6574E-01 & -0.1639E-01 & -0.1360E+01 & -0.2293E-01 & 0.9500E+00 & 0.1633E-01 & 0.4800E-02 & 0.4740E-02 \\ -0.1051E+01 & -0.3872E+01 & 0.1049E+01 & -0.5592E+01 & 0.8396E+00 & -0.3367E-00 & -0.1732E-07 & -0.6403E-09 \\ 0.9650E+00 & -0.5012E+00 & -0.2310E-01 & 0.1676E+00 & 0.1754E+01 & 0.5943E-01 & -0.1865E-00 & -0.1117E-07 \\ -0.3683E+00 & 0.5887E+01 & -0.3427E+00 & -0.1066E+01 & 0.3539E-01 & -0.1416E+01 & -0.1025E-07 & -0.5384E-09 \\ 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.1000E+01 & 0.7919E-03 & -0.2770E+01 & 0.0000E+00 & 0.0000E+00 \\ 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.9996E+00 & 0.2857E-01 & 0.0000E+00 & 0.0000E+00 \end{bmatrix}$$

Control Matrix

$$B = \begin{bmatrix} -0.0040756E-02 & -0.1719079E+00 \\ 0.1989993E+00 & -0.1195507E-01 \\ 0.1955618E+00 & -0.1342436E+01 \\ 0.9126846E+02 & -0.0747376E+00 \\ 0.1624011E+00 & 0.1937637E+02 \\ 0.1954920E+02 & -0.1055517E+01 \\ 0.0000000E+00 & 0.0000000E+00 \\ 0.0000000E+00 & 0.0000000E+00 \end{bmatrix}$$

Model at 330kph

System Matrix (8X8)

$$A = \begin{bmatrix} -0.6001E-01 & 0.7051E-02 & 0.0100E-01 & -0.4801E+02 & 0.1110E+00 & 0.3110E-02 & -0.4326E-11 & -0.1702E+00 \\ 0.4894E-02 & -0.3597E+00 & -0.1541E-01 & -0.1150E+00 & -0.0503E-02 & -0.1577E+01 & 0.1699E+00 & -0.1057E-02 \\ 0.1189E+00 & -0.5054E-01 & -0.1469E+01 & -0.4705E-01 & 0.1583E+01 & 0.2015E-07 & 0.9930E-02 & 0.1009E-01 \\ -0.9696E+00 & -0.7311E+01 & 0.5599E+01 & -0.4151E+01 & -0.1150E+01 & -0.1502E+00 & -0.2011E-07 & -0.9231E-09 \\ 0.5496E+00 & -0.1562E+00 & 0.5074E+01 & 0.5549E+00 & 0.2164E+01 & 0.7040E-01 & -0.1543E-06 & -0.1098E-07 \\ -0.1450E+01 & 0.7075E+01 & 0.3160E+01 & -0.4575E+00 & 0.4215E-01 & -0.2052E+01 & -0.1496E-07 & 0.1255E-09 \\ 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.1000E+01 & 0.6211E-02 & 0.1063E+00 & 0.0000E+00 & 0.0000E+00 \\ 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 0.9903E+00 & 0.5034E-01 & 0.0000E+00 & 0.0000E+00 \end{bmatrix}$$

Control Matrix

$$B = \begin{bmatrix} -0.4634399E-02 & -0.1160004E+00 \\ 0.2312920E+00 & 0.1221094E-01 \\ 0.4189311E+00 & -0.2252474E+01 \\ 0.8381099E+02 & 0.1546433E+02 \\ -0.1757970E+01 & 0.3070232E+02 \\ 0.1294223E+02 & 0.4747512E+01 \\ 0.0000000E+00 & 0.0000000E+00 \\ 0.0000000E+00 & 0.0000000E+00 \end{bmatrix}$$

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